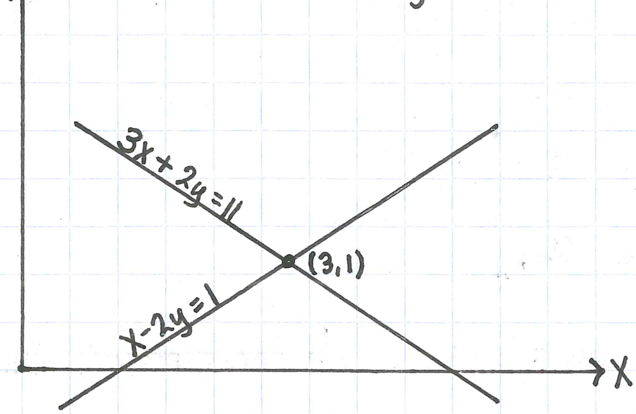
ESC103 - ENGINEERING MATHEMATICS and COMPUTATION

The central problem of linear algebra is to solve a set or system of linear equations.

What it means to "solve" a system of equations is to find values of x and y that satisfy all the equations in the system.

Consider Two Equations:

$$\rightarrow x - 2y = 1$$
$$\rightarrow 3x + 2y = 11$$



Row Picture

In the row picture, the equations refer to two straight lines, and the solution refers to the point of intersection of those two straight lines.

$$x\begin{bmatrix}1\\3\end{bmatrix} + y\begin{bmatrix}-2\\2\end{bmatrix} = \begin{bmatrix}1\\11\end{bmatrix}$$

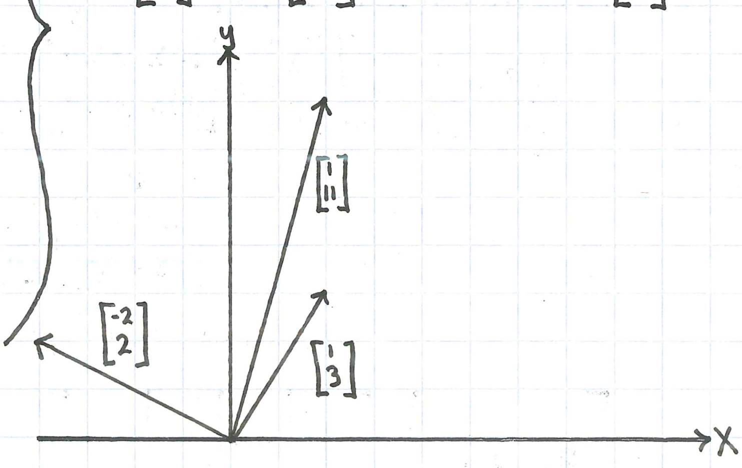
$$\begin{bmatrix}x\\3x\end{bmatrix} + \begin{bmatrix}-2y\\2y\end{bmatrix} = \begin{bmatrix}1\\11\end{bmatrix}$$

$$\begin{bmatrix}x-2y\\3x+2y\end{bmatrix} = \begin{bmatrix}1\\11\end{bmatrix}$$

$$x - 2y = 1$$
$$3x + 2y = 11$$

Column Picture

In the column picture, we are looking for linear combinations of the vectors $\begin{bmatrix}1\\3\end{bmatrix}$ and $\begin{bmatrix}-2\\2\end{bmatrix}$ that produce $\begin{bmatrix}1\\11\end{bmatrix}$



$$3\begin{bmatrix}1\\3\end{bmatrix} + 1\begin{bmatrix}-2\\2\end{bmatrix} = \begin{bmatrix}1\\11\end{bmatrix}$$

What is a Matrix?

A matrix is a rectangular array of numbers.

Example:

$$A = \begin{array}{cc} \text{3 columns} & \\ \begin{bmatrix} 4 & 8 & 3 \\ 2 & 1 & -9 \end{bmatrix} & \text{2 rows} \end{array}$$

Matrix A has 2 rows and 3 columns
A is a 2×3 (2 by 3) matrix

To denote an entry in matrix A, we say

a_{ij} Where i denotes the row
j denotes the column

Example:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

Recall $x - 2y = 1$ and $3x + 2y = 11$. We can represent this as a matrix:

$$\underbrace{\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}}_{\text{Matrix } A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\vec{v}} = \underbrace{\begin{bmatrix} 1 \\ 11 \end{bmatrix}}_{\vec{b}} \quad \Bigg\} \quad A\vec{v} = \vec{b}$$

$A\vec{v}$ (Matrix times vector)

$$A\vec{v} = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (1)x & (-2)y \\ (3)x & (+2)y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

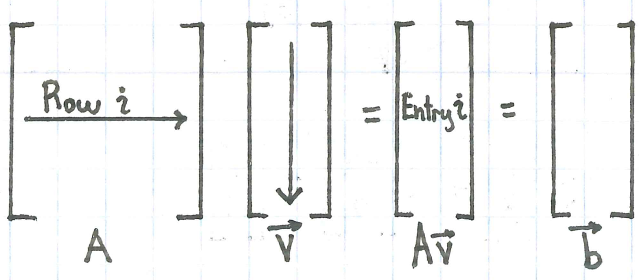
$$\downarrow$$

$$\begin{bmatrix} x - 2y \\ 3x + 2y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

Row Picture

$\therefore$ The first entry of $A\vec{v}$ is the dot product of the first row of A with $\vec{v}$

The second entry of $A\vec{v}$ is the dot product of the second row of A with $\vec{v}$



Entry i in $A\vec{v}$ = Dot product of Row i of Matrix A with column vector $\vec{v}$

For the dot product to be something we can actually calculate, there have to be the same amount of entries in row i as there are in $\vec{v}$. Otherwise it is not defined.

The number of columns in matrix A determines the number of entries in row i.

$$A\vec{v} = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = x\begin{bmatrix} 1 \\ 3 \end{bmatrix} + y\begin{bmatrix} -2 \\ 2 \end{bmatrix} \Bigg\} \text{ Column Picture}$$

$$\begin{bmatrix} x \\ 3x \end{bmatrix} + \begin{bmatrix} -2y \\ 2y \end{bmatrix} = \begin{bmatrix} x-2y \\ 3x+2y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$$

Summary

- → Scalar Times A Vector $c\vec{v}$
- → Vector Times A Vector $\vec{v} \cdot \vec{w}$ $\vec{v} \times \vec{w}$
- → Matrix Times A Vector $A\vec{v}$