ESC103 - ENGINEERING MATHEMATICS and COMPUTATION

Multiplying Vectors Using the Cross Product

Let $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$

Definition of the Cross Product (Vector Product)

$$\vec{u} = \vec{v} \times \vec{w} = \begin{bmatrix} v_2 w_3 - v_3 w_2 \\ -v_1 w_3 + v_3 w_1 \\ v_1 w_2 - v_2 w_1 \end{bmatrix}$$

Prove $\vec{u}$ is orthogonal to both $\vec{v}$ and $\vec{w}$ Proof Assigned

Hint: $\vec{u} \cdot \vec{v} = 0$

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Properties of the Cross Product

P1: $\vec{v} \times (\vec{w} + \vec{z}) = \vec{v} \times \vec{w} + \vec{v} \times \vec{z}$

- Distributive Law

P2: $\vec{v} \times \vec{w} = -\vec{w} \times \vec{v}$

- Anti Commutative Law

P3: $\vec{v} \times \vec{0} = \vec{0} \times \vec{v} = \vec{0}$

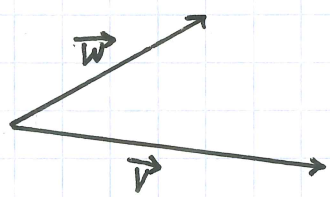
- Commutative Only with $\vec{0}$

P4: $c(\vec{v} \times \vec{w}) = (c\vec{v}) \times \vec{w} = \vec{v} \times (c\vec{w})$

Direction: Right Hand Rule

$\vec{v} \times \vec{w}\ \odot$

$\vec{w} \times \vec{v}\ \otimes$



Magnitude: Lagrange Identity

$$\|\vec{v} \times \vec{w}\|^2 = \|\vec{v}\|^2 \|\vec{w}\|^2 - (\vec{v} \cdot \vec{w})^2 \quad (1)$$

Recall $\vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos\theta \quad (2)$

Sub (2) into (1)

$$\|\vec{v} \times \vec{w}\|^2 = \|\vec{v}\|^2 \|\vec{w}\|^2 - \|\vec{v}\|^2 \|\vec{w}\|^2 \cos^2\theta$$

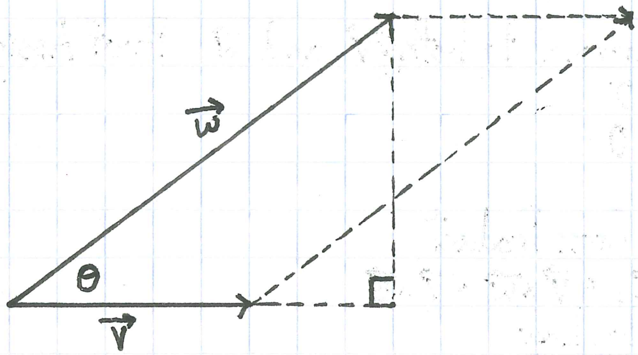
$$\|\vec{v}\times\vec{w}\|^2 = \|\vec{v}\|^2\|\vec{w}\|^2(1-\cos^2\theta)$$

Recall $\sin^2\theta + \cos^2\theta = 1$

$$\|\vec{v}\times\vec{w}\|^2 = \|\vec{v}\|^2\|\vec{w}\|^2\sin^2\theta$$

$$\therefore \|\vec{v}\times\vec{w}\| = \|\vec{v}\|\|\vec{w}\|\sin\theta$$

Application of the Cross Product (Area of a parallogram)



$$\text{Area} = \|\vec{v}\|\|\vec{w}\|\sin\theta$$
$$= \|\vec{v}\times\vec{w}\|$$