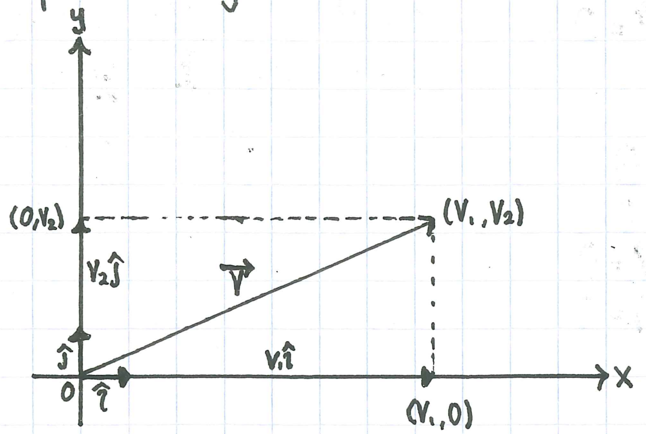
ESC103 - ENGINEERING MATHEMATICS and COMPUTATION

Projecting One Vector on Another Vector

Concept of Projection:



What is V_1?

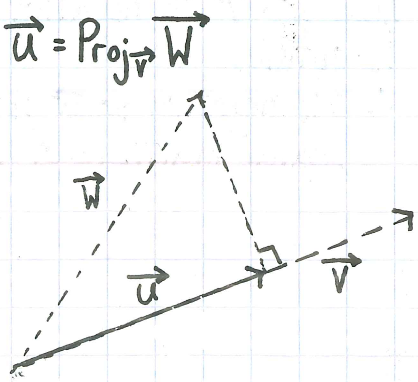
V_1 is found by projecting $\vec{V}$ on $\hat{\imath} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

By vector addition:

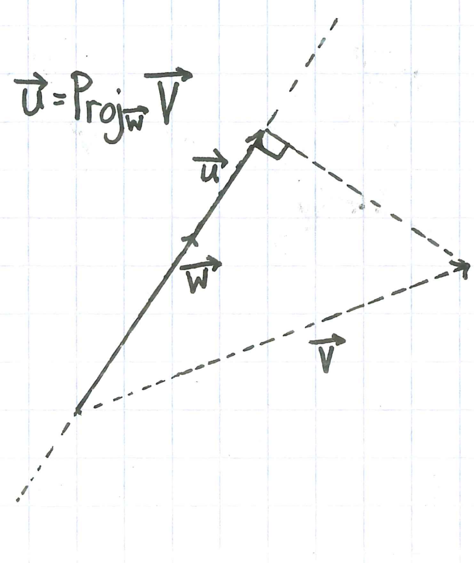
$$\begin{aligned} \vec{V} &= V_1\hat{\imath} + V_2\hat{j} \\ &= V_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + V_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} V_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ V_2 \end{bmatrix} \\ &= \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \end{aligned}$$

Let's generate the concept of projection to the projection of one vector $(\vec{W})$ on another vector $(\vec{V})$.

$\vec{u} = \text{Proj}_{\vec{V}}\vec{W}$



$\vec{u} = \text{Proj}_{\vec{W}}\vec{V}$



Next, let's develop an expression for $\vec{u}$ based on knowledge of $\vec{v}$ and $\vec{w}$.
Based on the way we have constructed $\vec{u}$, it has two properties:

- P1: $\vec{u}$ is parallel to $\vec{v}$, it can be expressed as a scalar multiple of $\vec{v}$
  $\therefore \vec{u} = c\vec{v}$ where c is an unknown scalar
- P2: $\vec{w} - \vec{u}$ (and $\vec{u} - \vec{w}$) is orthogonal to $\vec{v}$
  $\therefore (\vec{w} - \vec{u}) \cdot \vec{v} = 0$

Solve for c

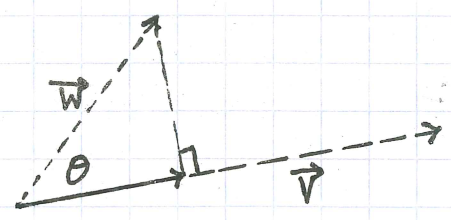
$$(\vec{w} - \vec{u}) \cdot \vec{v} = 0$$
$$\vec{w} \cdot \vec{v} - \vec{u} \cdot \vec{v} = 0$$
$$\vec{w} \cdot \vec{v} - (c\vec{v}) \cdot \vec{v} = 0$$
$$\vec{w} \cdot \vec{v} - c(\vec{v} \cdot \vec{v}) = 0$$

$$\therefore c = \frac{\vec{w} \cdot \vec{v}}{\vec{v} \cdot \vec{v}}$$

$$\therefore \boxed{\vec{u} = \frac{\vec{w} \cdot \vec{v}}{\vec{v} \cdot \vec{v}} \vec{v}}$$

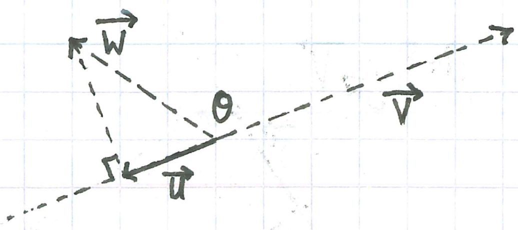
Note: The numerator can be positive or negative but the denominator is always positive.

Example:



Here $\vec{w} \cdot \vec{v}$ is positive because θ is acute.
$\therefore c$ is positive so $\vec{u}$ is a positive scalar multiple of $\vec{v}$.

Example:



Here $\vec{w} \cdot \vec{v}$ is negative because θ is obtuse.
$\therefore c$ is negative so $\vec{u}$ is a negative scalar multiple of $\vec{v}$.