ESC103-ENGINEERING MATHEMATICS and COMPUTATION

Length of a Vector (Magnitude of a Vector)

$$\left.\begin{array}{l} R^2 \quad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \|\vec{v}\| = \sqrt{v_1^2 + v_2^2} \\ \\ R^3 \quad \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad \|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2} \end{array}\right\} \text{Extends to } R^n$$

Example:

$$\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \|\vec{v}\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

Properties of Vector Addition

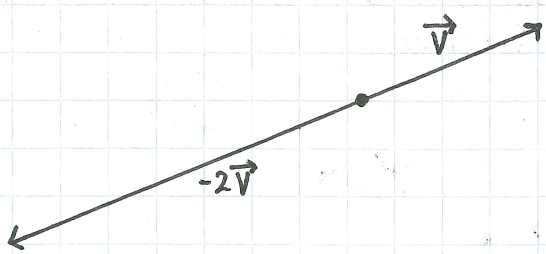
P1: $\vec{v} + \vec{w} = \vec{w} + \vec{v}$
- Commutative Law

P2: $(\vec{v} + \vec{w}) + \vec{z} = \vec{v} + (\vec{w} + \vec{z})$
- Associative Law

Properties of Scalar Multiplication

Question: $\|c\vec{v}\| = c\|\vec{v}\|$? NO!

Example:



P1: $\|c\vec{v}\| = |c|\|\vec{v}\|$

Note: $\|\vec{v}\| = 0 \Leftrightarrow \vec{v} = \vec{0}$

Unit Vector

Famous unit vectors $(\hat{\imath}, \hat{\jmath}, \hat{k})$ lie on (x, y, z) $\quad \hat{\imath} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \hat{\jmath} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
- Unit vectors have length 1

You can turn any vector into a unit vector by normalizing it (dividing by its magnitude)

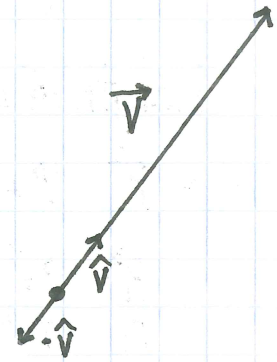
Example:

Recall: $\|\vec{v}\| = \sqrt{14}$

$$\left\| \frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\| = \left| \frac{1}{\sqrt{14}} \right| \left\| \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} \right\| = \frac{1}{\sqrt{14}} \cdot \sqrt{14} = 1$$

Unit Vector

$$\frac{1}{\sqrt{14}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{14}} \\ \frac{2}{\sqrt{14}} \\ \frac{3}{\sqrt{14}} \end{bmatrix}$$

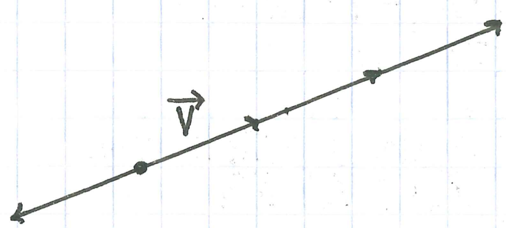


Vector Spaces

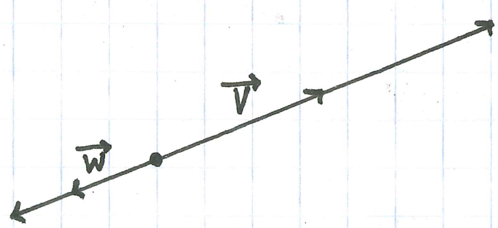
Suppose we have 3 vectors $\vec{v}, \vec{w}, \vec{z}$ in R^3 and scalars c, d, e.

Consider all combinations of $c\vec{v}$ if $\vec{v} = \vec{0} \Rightarrow$ all possible combinations of $c\vec{v} = \vec{0}$
- Zero dimensional vector space

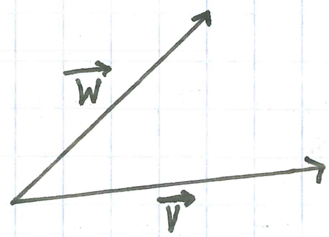
Consider all combinations of $c\vec{v}$ if $\vec{v} \neq \vec{0}$ (line)
- one dimensional vector space



Consider all combinations of $c\vec{v} + d\vec{w}$ if $\vec{v} \parallel \vec{w}$ (line)
- One dimensional vector space

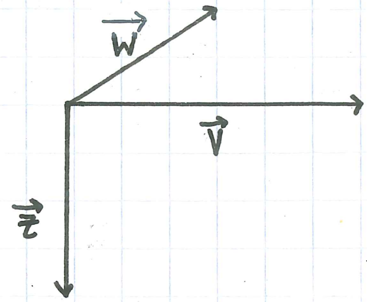


Consider all combinations of $c\vec{v} + d\vec{w}$ if $\vec{v}$ is not $\parallel$ to $\vec{w}$ (Plane)
- two dimensional vector space



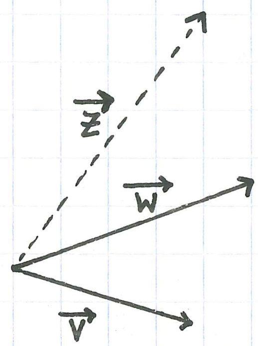
Consider all combinations of $c\vec{v} + d\vec{w} + c\vec{z}$ if $\vec{v}$, $\vec{w}$, and $\vec{z}$ are on the same plane.

- two dimensional vector space
- redundancy of the third vector (related to dependancy)



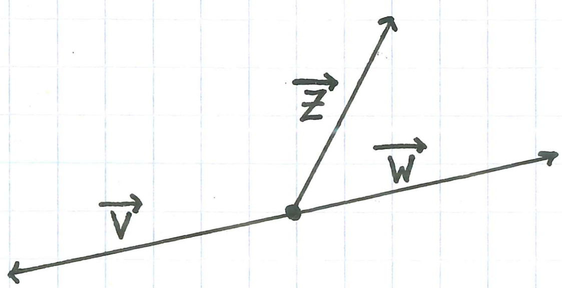
Consider all combinations of $c\vec{v} + d\vec{w} + e\vec{z}$ if $\vec{v}$ and $\vec{w}$ are on the same plane and $\vec{z}$ is on a different plane.

- three dimensional vector space
- can move anywhere in R^3
- you need all three vectors (related to independancy)



Consider all combinations of $c\vec{v} + d\vec{w} + e\vec{z}$ if $\vec{v} \parallel \vec{w}$ and $\vec{v}$, $\vec{w}$, and $\vec{z}$ are in the same plane.

- redundancy of one of $\vec{v}$ and $\vec{w}$ to move in a plane with $\vec{z}$



Note: The number of vectors does not accurately represent the number of dimensions.