ESC103 - ENGINEERING MATHEMATICS and COMPUTATION

Vector - A quantity with a magnitude, direction, and sometimes units (Physics). A vector is defined by its magnitude and direction.

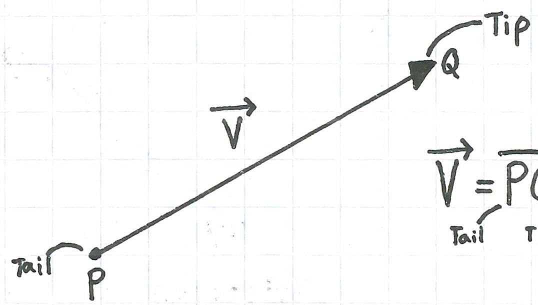
Scalar - A quantity with a magnitude and sometimes units (physics)
- a scalar has a positive and negative direction

A scalar is defined by its magnitude.

Line - A collection of points that go to ∞ in both directions. The line is different if its position is moved.

At the heart of linear algebra are two operations. These are adding vectors and multiplying vectors by scalars.

How To Draw A Vector:



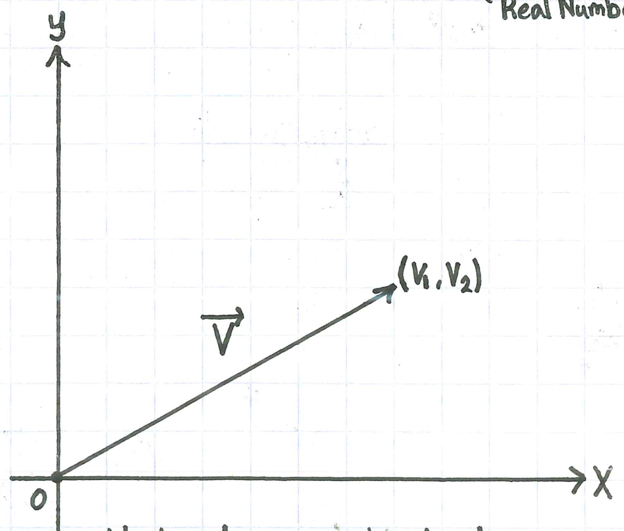
$\vec{V} = \overrightarrow{PQ}$ (P: Tail, Q: Tip)

Note: Order is important $\overrightarrow{PQ} \neq \overrightarrow{QP}$

Dimension

This vector is drawn in R^2 (2-D)

Real Numbers



Vector drawn in standard position

Tail is at the origin
Tip is at (V_1, V_2)
} Standard Position

$\vec{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ Column Vector

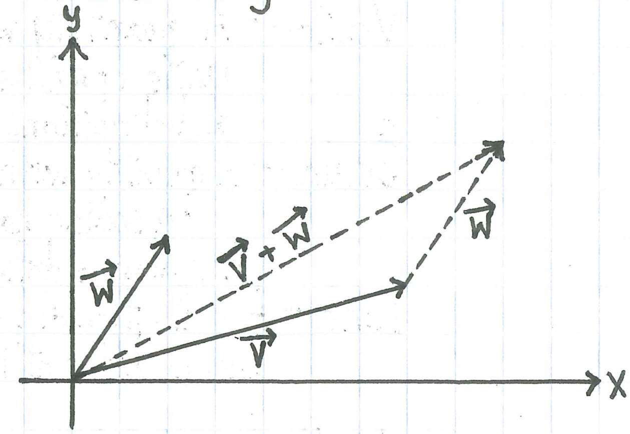
Vector Addition

Algebraically

$\vec{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \vec{W} = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}$

$\vec{V} + \vec{W} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} V_1 + W_1 \\ V_2 + W_2 \end{bmatrix}$

Geometrically

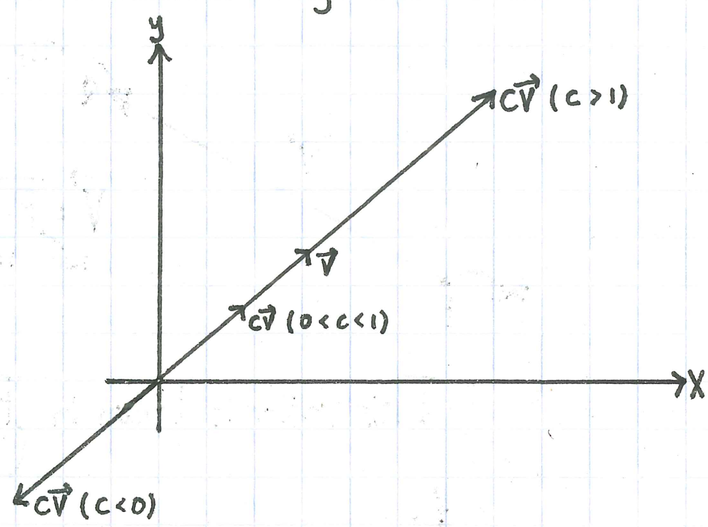


Scalar Multiplication

Algebraically

$C\vec{V} = \begin{bmatrix} CV_1 \\ CV_2 \end{bmatrix}$ where c is a scalar

Geometrically



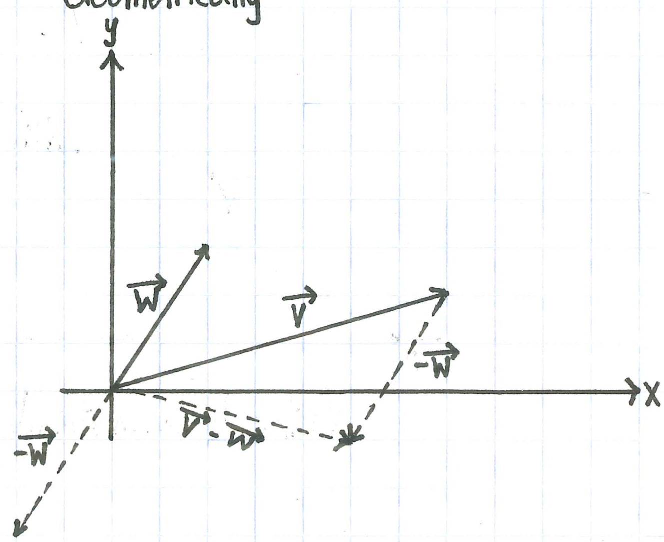
Vector Subtraction

Algebraically

$\vec{V} - \vec{W} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} - \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \begin{bmatrix} V_1 - W_1 \\ V_2 - W_2 \end{bmatrix}$

$\vec{V} - \vec{W} = \vec{V} + (-1)\vec{W}$

Geometrically



Zero Vector

$$\vec{v} - \vec{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \vec{0}$$

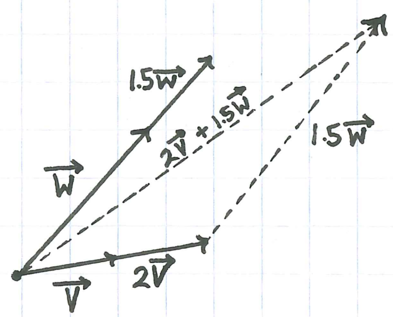
Linear Combinations

Let c and d be scalars:

$$c\vec{v} + d\vec{w}$$

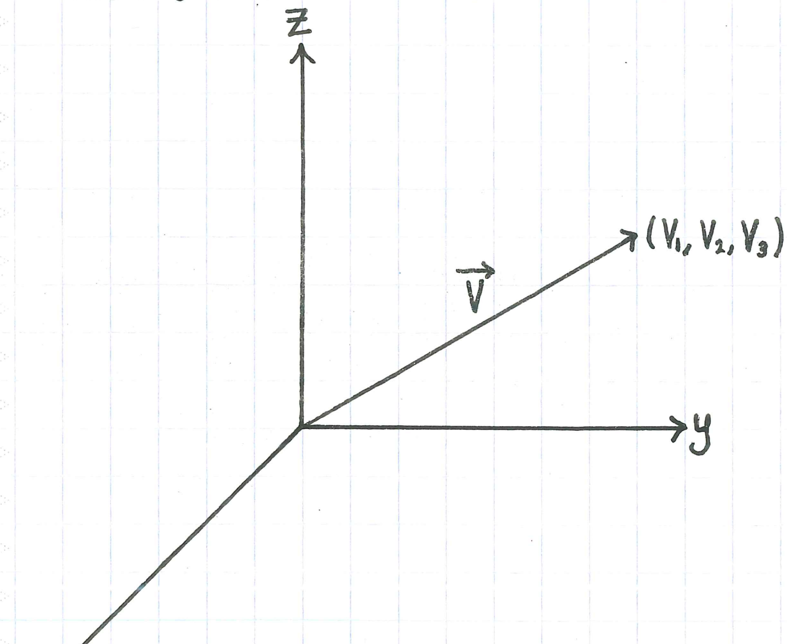
is a linear combination of $\vec{v}$ and $\vec{w}$.

Example: $2\vec{v} + 1.5\vec{w}$



This is a linear combination of $\vec{v}$ and $\vec{w}$

Vectors in R^3



$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\underbrace{\underset{\text{Column Vector } (3 \times 1)}{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}} \neq \underset{\text{Row Vector } (1 \times 3)}{\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}}}_{\text{Dimensionally Not Equal}}$$

Transpose

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}^T = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$$