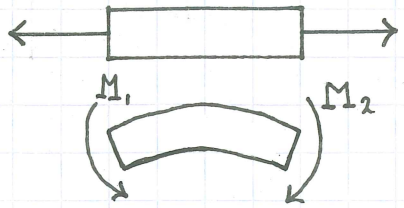
CIV102 - STRUCTURES and MATERIALS

Topic: What is I?

1) So far



Statics

$\Sigma F_x = 0$

$\Sigma M = 0$

Dynamics

$F = ma$ — linear acceleration $\left[\frac{m}{s^2}\right]$

mass

$M = I_m \cdot \alpha$ — rotational acceleration $\left[\frac{rad}{sec^2}\right]$

I_m: Mass moment of Inertia

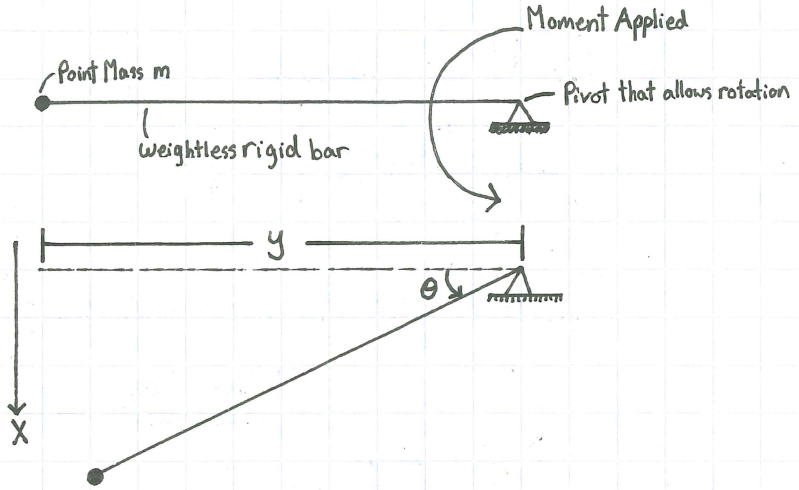
Mass moment of Inertia = I_m

Second Moment of Area = I

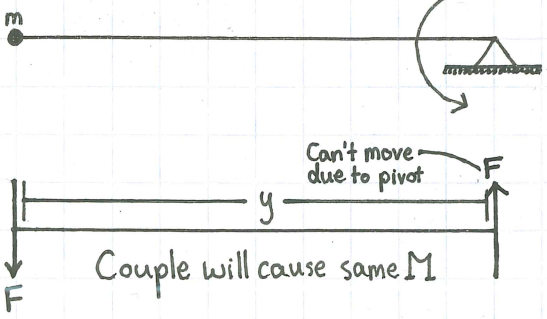
2) I_m = ?

Assumptions:

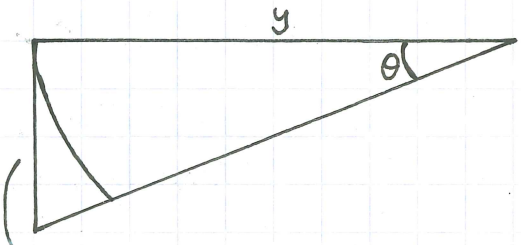
- Mass is a point mass
- Angles are small
- One mass



Forces



Geometry



$\cong \theta \cdot y$ = arclength which almost equals linear displacement

$M = F \cdot y$

$F = \frac{M}{y}$

	Variable	Linear Distance	Linear Velocity	Linear Acceleration
Linear	X	X	$\frac{dx}{dt}$	$\frac{d^2x}{dt^2}$
	θ	θy	$y\frac{d\theta}{dt}$	$y\frac{d^2\theta}{dt^2}$

Apply $F = ma$

$$\frac{M}{y} = my\frac{d^2\theta}{dt^2}$$

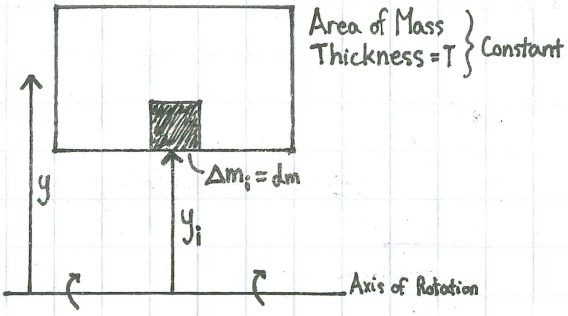
$$M = m \cdot y^2 \cdot \frac{d^2\theta}{dt^2} = \alpha$$

$$M = (my^2)\alpha$$

3) Add more masses

$$\boxed{I_m = \sum_{i=1}^{n} m_i \cdot y_i^2}$$

4) Masses with volume (not point masses)



$$I_m = \sum m_i \cdot y_i^2$$

take limit as $\Delta m_i \longrightarrow 0$

$$\boxed{I_m = \int y^2\, dm}$$

5) Second Moment of Area = I

· Property of a cross section

Lets define ρ = mass density of material

Mass of Object $= \rho \cdot \text{Area} \cdot t$

$$I_m = \int y^2\, dm \quad (dA = \text{Area})$$

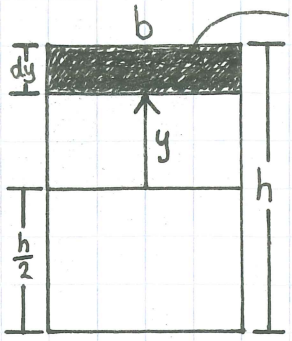
$$= \int y^2 \cdot \rho \cdot t\, dA$$

$$I_m = \rho \cdot t \boxed{\int y^2\, dA}$$

define this as second moment of area; I [mm^4] or moment of inertia

$$I_m = \rho \cdot t \cdot I$$

6) I for a rectangle rotating about its centroid



$$I = \int y^2 \, dA$$

$$= \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 \, b \cdot dy$$

$$I = \frac{by^3}{3}\Big|_{-\frac{h}{2}}^{\frac{h}{2}} = \left[\frac{b \cdot h^3}{2^3 \cdot 3} - \left(b\left(\frac{-h}{2}\right)^3 \cdot \frac{1}{3}\right)\right]$$

$$\boxed{I = \frac{bh^3}{12}}$$