CIV102 - STRUCTURES and MATERIALS

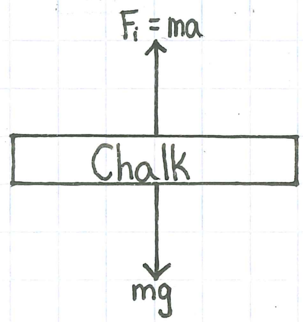
Topic: Oscillating Bodies

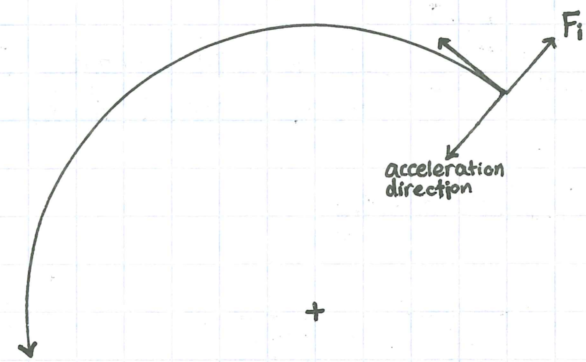
1) Oscillating Systems

Mechanical Oscillating Systems
Electrical Oscillating Systems
Electromagnetic Oscillating Systems

2) Dynamic Equilibrium

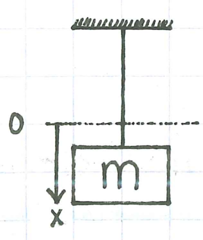
D'Alembert's Principle: "An accelerating system where the sum of the forces does not equal zero, can be placed in dynamic equilibrium by introducing an Inertial Force $= F_i = ma$ acting in a direction opposite to acceleration direction.





3) Free Vibration

·Ignore Gravity



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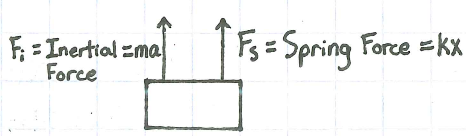


$$\sum F_x = 0$$
$$0 = -F_s - F_i$$
$$= F_s + F_i$$
$$0 = kx + mg$$

Assume is accelerating downwards.

$$\text{Velocity} = \frac{dx}{dt} = V$$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = a$$

$$0 = kx + m\frac{d^2x}{dt^2}$$ differential equation

guess + Check

$$X(t) = \overset{\text{Scale Factor}}{A}\sin(\underbrace{\omega_n}_{\text{Natural Frequency}} t + \overset{\text{Phase Shift}}{\phi})$$

$$\frac{dx}{dt} = A\omega_n\cos(\omega_n t + \phi)$$

$$\frac{d^2x}{dt^2} = -A\omega_n^2\sin(\omega_n t + \phi)$$

Substitute into differential equation

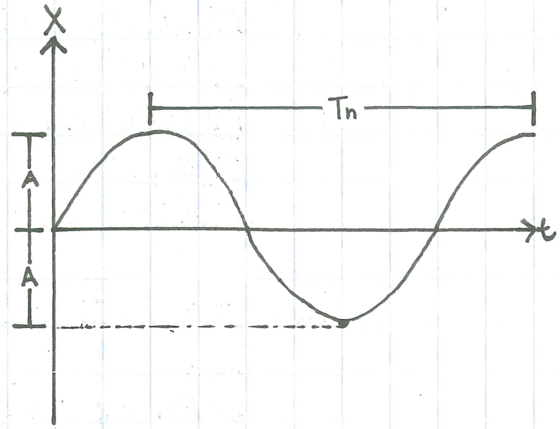
$$0 = kx + m\frac{d^2x}{dt^2}$$
$$0 = k\cdot A\sin(\omega_n t + \phi) - m\cdot A\omega_n^2\sin(\omega_n t + \phi)$$
$$0 = k - m\omega_n^2$$

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}}$$

ω_n = natural frequency $\left[\frac{rad}{sec}\right]$
└ rate it wants to vibrate at

Convert $\omega_n\left[\frac{rad}{sec}\right]$ into $f_n = Hz = \left[\frac{Cycles}{Second}\right]$

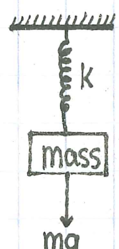
$$f_n = \frac{1}{2\pi}\cdot\omega_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



$$T_n = \text{Natural Period} = \frac{1}{f_n} = 2\pi\sqrt{\frac{m}{k}}$$

$$\boxed{T_n = 2\pi\sqrt{\frac{m}{k}}}$$

4) Add in gravity



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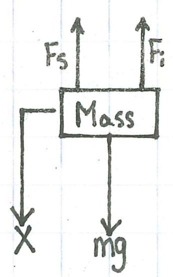


$$\Sigma F_x = 0$$
$$0 = mg - F_i - F_s$$
$$0 = m\frac{d^2x}{dt^2} + kx - mg$$

guess + Check

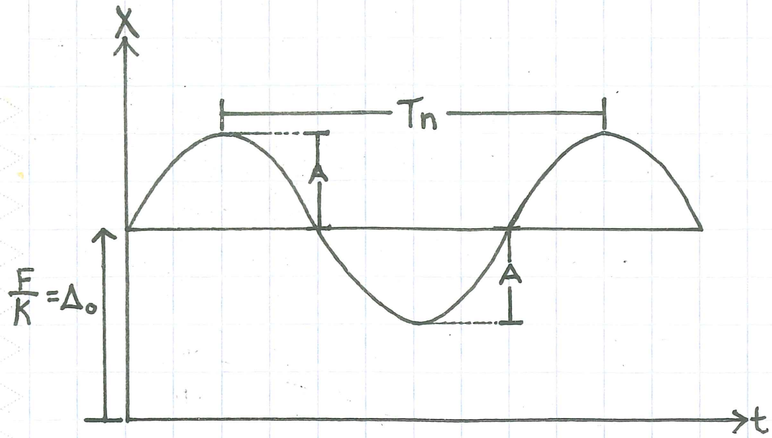
guess $X(t) = A\sin(\omega_n t + \phi) + \Delta_o$

Δ_o = Static displacement caused by load mg

$$F = k\Delta_o$$

$$\boxed{\Delta_o = \frac{F}{k}}$$

Plot It



Simpler form for gravity-involved forces

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad \Delta_o = \frac{mg}{K}, \quad K = \frac{mg}{\Delta_o}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{\cancel{m}g}{\Delta_o \cancel{m}}} \qquad \text{if } \Delta_o \text{ in mm, } g = 9810\,\frac{mm}{s^2}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9810}{\Delta_o}}$$

$$\boxed{f_n = \frac{15.76}{\sqrt{\Delta_o}}}$$

Demonstration $\quad \Delta_o = 320\,mm$

$$f_n = \frac{15.76}{\sqrt{320\,mm}} = 0.88\,Hz \quad \text{(Theoretical)}$$

$$\frac{10 \text{ cycles}}{11.7} = 1.171\,\frac{sec}{cycles}$$
$$= 0.854\,Hz \text{ (Experimental)}$$