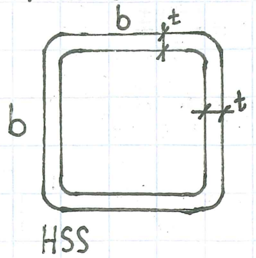
CIV102 - STRUCTURES and MATERIALS

Topic: Thin Plate Buckling

1) Thin Wall Members

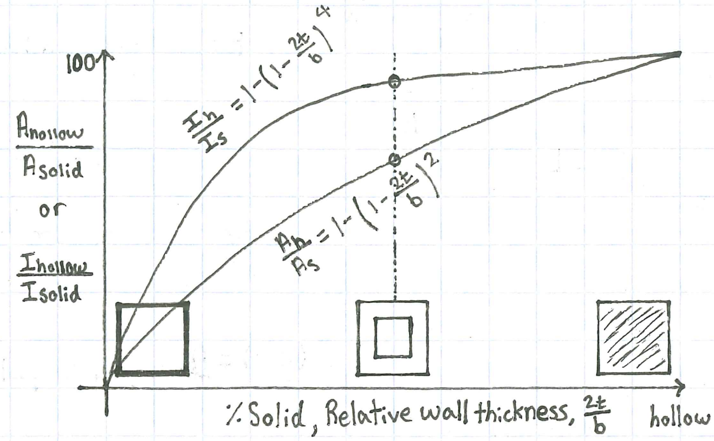


$$Area = b^2 - (b-2t)^2 \qquad I = \frac{b^4}{12} - \frac{(b-2t)^4}{12}$$

$$\frac{A_{hollow}}{A_{solid}} = \frac{b^2-(b-2t)^2}{b^2} = 1-\left(1-\frac{2t}{b}\right)^2$$ (weight related to this)

$$\frac{I_{hollow}}{I_{solid}} = \frac{\frac{b^2}{12} - \frac{(b-2t)^4}{12}}{\frac{b^4}{12}} = 1-\left(1-\frac{2t}{b}\right)^4$$ (Strength related to this)

Strength = quartic



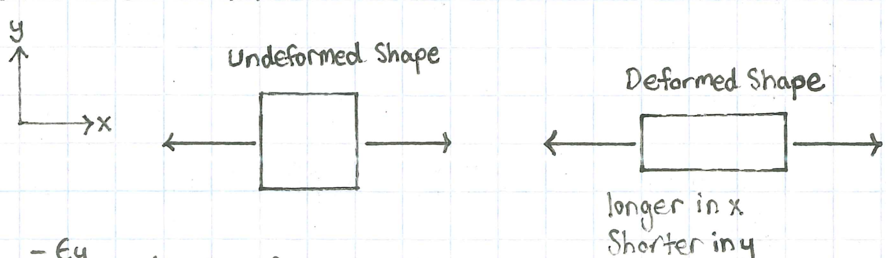
$t = \frac{b}{2} \longrightarrow$ Empty

Thinner walled members have better strength to weight ratios

2) Plate Buckling

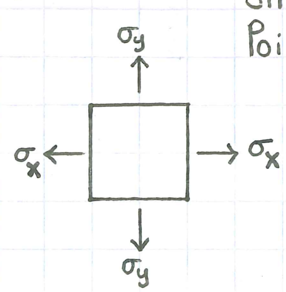
- Will limit how thin the wall can be
- Derivation is hard

2a) Poisson's Ratio



$$\frac{-\epsilon_y}{\epsilon_x} = \mu = \nu$$ Poisson's ratio is constant

rubber, $\mu = 0.5$
Steel, $\mu = 0.3$
Cork, $\mu = 0$



Strain from Stress adds to strain from Poisson's Ratio

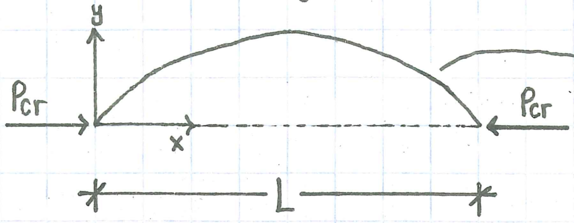
$$\epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} \qquad \epsilon_y = \frac{-\mu\sigma_x}{E} + \frac{\sigma_y}{E}$$

$$\sigma_x = \frac{E}{1-\mu^2}(\epsilon_x + \mu\epsilon_y)$$

$$\sigma_y = \frac{E}{1-\mu^2}(\mu\epsilon_x + \epsilon_y)$$

$\frac{E}{1-\mu^2}$ ← Effective E?

2b) Euler Buckling

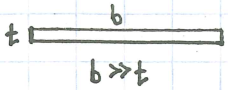


$$y(x) = A\sin\left(\frac{n\pi x}{L}\right) \quad \leftarrow n \text{ integer}$$

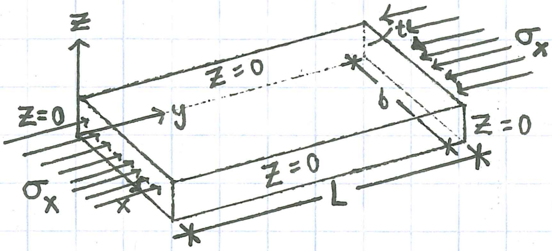
$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

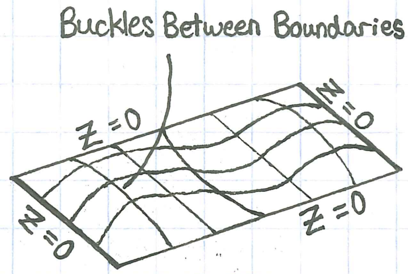
$$\frac{P_{cr}}{A} = \frac{\pi^2 E \cancel{b} t^{\cancel{3}}}{12 L^2 \cancel{bt}}$$

$$\sigma_{crit} = \frac{\pi^2 E}{12}\left(\frac{t}{L}\right)^2$$ Global Buckling/ Euler Critical Stress



2 c) Local Buckling Equation



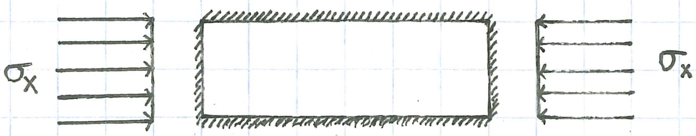


$$z(x,y) = A\sin\left(\frac{mx\pi}{L}\right)\sin\left(\frac{ny\pi}{L}\right)$$ m,n are 1,2,3,...

3) Thin Plate Buckling Equation

$$\sigma_{cr} = \frac{K\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2$$ if $L < b$ use $\left(\frac{t}{L}\right)^2$

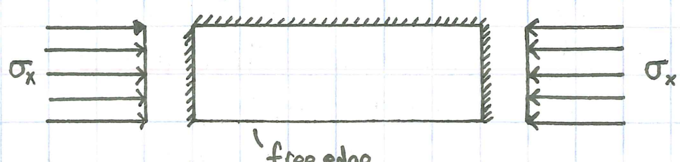
Case 1 – Restrained Against z Displacements on all 4 Sides

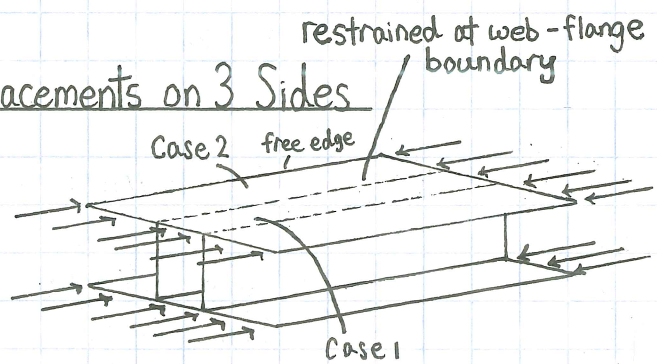


$K = 4$

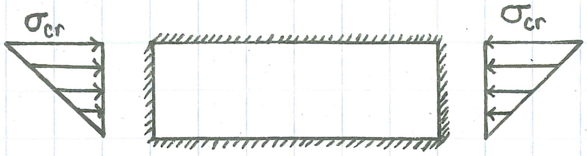
Case 2 – Restrained Against z Displacements on 3 Sides

$K = 0.425$



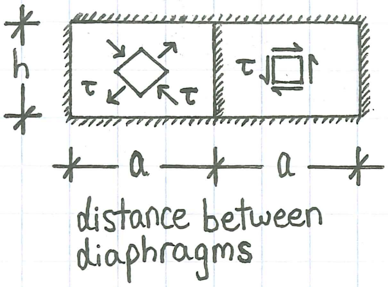


Case 3



$K = 6$

Case 4 – Shear Buckling



$$\tau_{cr} = \frac{5\pi^2 E}{12(1-\mu^2)}\left[\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right]$$