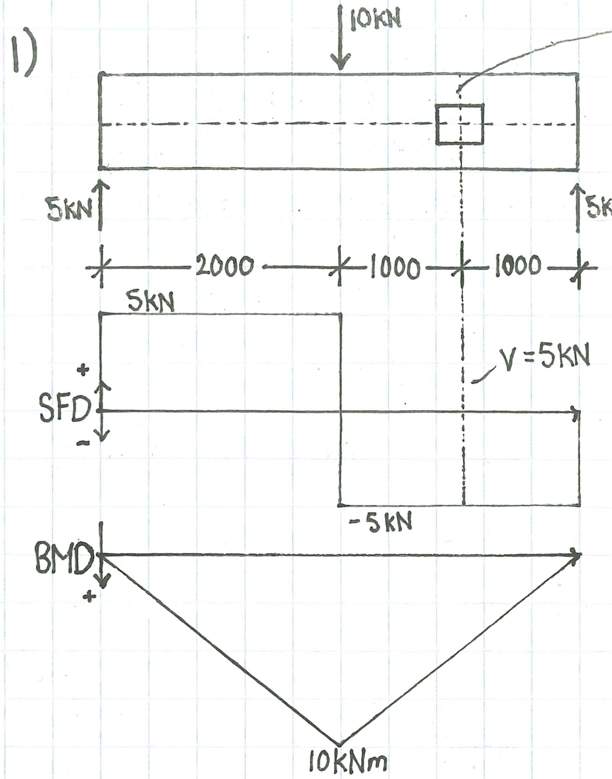
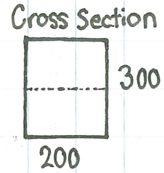
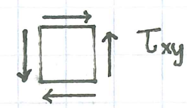
CIV102 - STRUCTURES and MATERIALS

Topic: τ_{xy} & $\frac{VQ}{IB}$

1)







$$\tau_{xy} = \frac{VQ}{IB}$$

V = Shear Force (N)

I = Second Moment of Area (mm^4)

b = Width of beam @ depth of interest

Q, First Moment of Area
= Geometric parameter to allow τ_{xy} to be calculated
⟶ depends on depth of interest

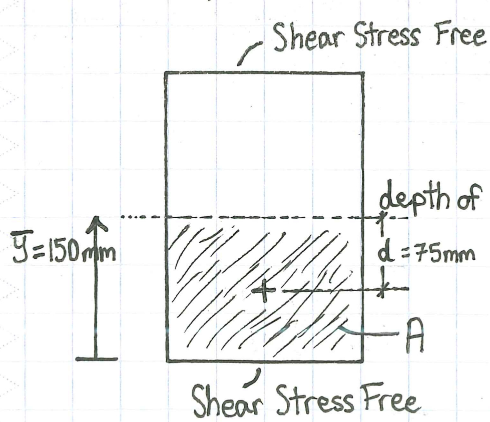
$Q = A \cdot d$

A = area between point of interest and a shear stress free surface

d = distance from the centroid of the area of interest to the centroid of the entire cross section

1st moment of area between a shear stress free surface (usually top or bottom) and the depth of interest with lever arm, d, taken as distance from centroid of area to centroidal axis of whole cross section, $\bar{y}$

Example Case Calculate τ_{xy} @ Midheight



$$\tau_{xy} = \frac{VQ}{IB}$$

$$I = \frac{bh^3}{12} = 450 \times 10^6\ mm^4$$

$V = 5000N \quad b = 200mm$

$$Q = Ad = 200 \cdot 150 \cdot 75$$
$$= 2.25 \times 10^6\ mm^3$$

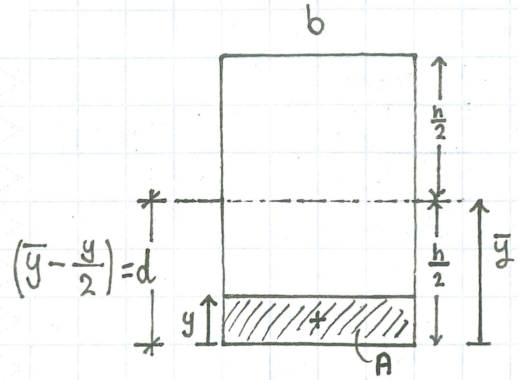
$$\tau_{xy} @ \bar{y} = \frac{VQ}{IB} = \frac{5000 \cdot 2.25 \times 10^6}{450 \times 10^6 \cdot 200} = 0.125 MPa$$

* Design for max τ_{xy} Stress

Q is a maximum @ $\bar{y}$ $\therefore$ $\tau_{xy\,max} = 0.125 MPa$ here

$$\text{Average Shear Stress} = \frac{V}{bh} = \frac{5000}{200 \cdot 300} = 0.0833 MPa < \tau_{xy\,max}$$

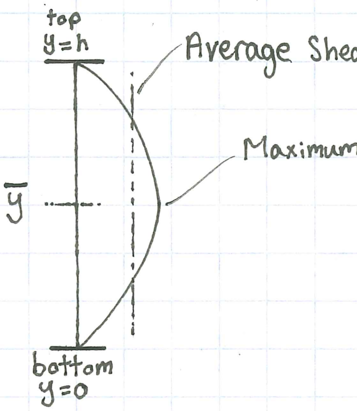
2) Variation of τ_{xy} vs. Depth



$$Q = A \cdot d$$
$$= by \cdot \left(\bar{y} - \frac{y}{2}\right)$$
$$= by \cdot \left(\frac{h}{2} - \frac{y}{2}\right)$$
$$= \frac{by}{2}(h-y) \longrightarrow \text{Parabolic with depth}$$

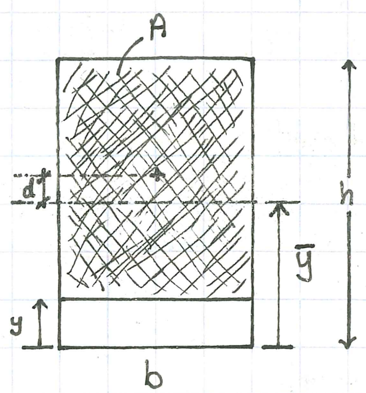
$$\tau_{xy} = \frac{VQ}{IB}$$
$$= \frac{V \cdot \cancel{b}y(h-y)12}{2bh^3 \cancel{b}}$$
$$= \text{Something!}$$

Plot τ_{xy}



$$\tau_{xy} @ \frac{h}{2},\ y = \frac{h}{2}$$
$$\tau_{xy} = \frac{Vhh12}{222bh^3}$$
$$\tau_{xy} = \frac{3V}{2bh}$$

Redo with calculation of Q from top

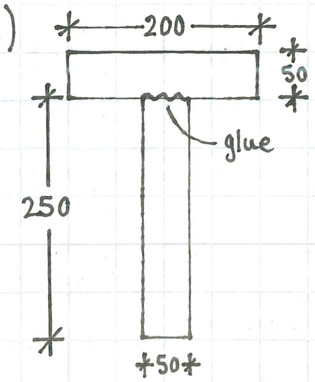


$$d = \frac{h-y}{2} + y - \bar{y}$$
$$= \frac{h}{2} - \frac{y}{2} + y - \frac{h}{2}$$
$$d = \frac{y}{2}$$

$$Q = A \cdot d$$
$$= b(h-y)\frac{y}{2}$$

Same as from bottom

3)



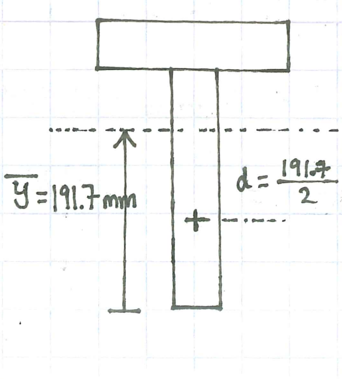
i) $\tau_{xy\,max}$?

ii) τ_{xy} @ glue joint

$\bar{y} = 191.7\text{mm}$ $\quad I = 192.2 \times 10^6\ \text{mm}^4$

$V = 40\text{kN}$

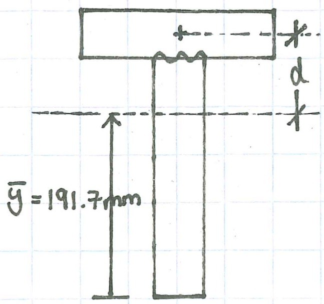
i) τ_{xy} will be max @ $\bar{y}$



$$d = \frac{191.7}{2} = 95.85\text{mm}$$

$$Q = A \cdot d$$
$$= 191.7 \cdot 50 \cdot \frac{191.7}{2}$$
$$= 918.7 \times 10^3\ \text{mm}^3$$
$$\tau_{xy} = \frac{40000 \cdot 918.7 \times 10^3}{192.2 \times 10^6 \cdot 50}$$
$$\underline{\tau_{xy\,max} = 3.82\text{MPa}}$$

ii) τ_{xy} on glue joint



$$d = 250 - \bar{y} + 25 \Rightarrow d = 83.3\,mm$$

$$Q = 200 \cdot 50 \cdot 83.3 \Rightarrow Q = 833 \times 10^3\,mm^3$$

$$\tau_{xy,glue} = \frac{VQ}{IB} = \frac{40000 \cdot 833 \times 10^3}{192.2 \times 10^6 \cdot 50}$$

$$\underline{\underline{\tau_{xy,glue} = 3.47\,MPa}}$$

τ_{xy} Distribution