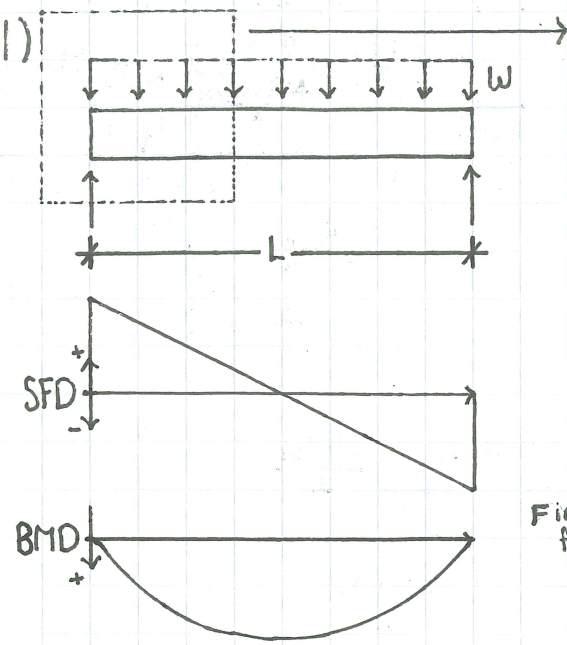
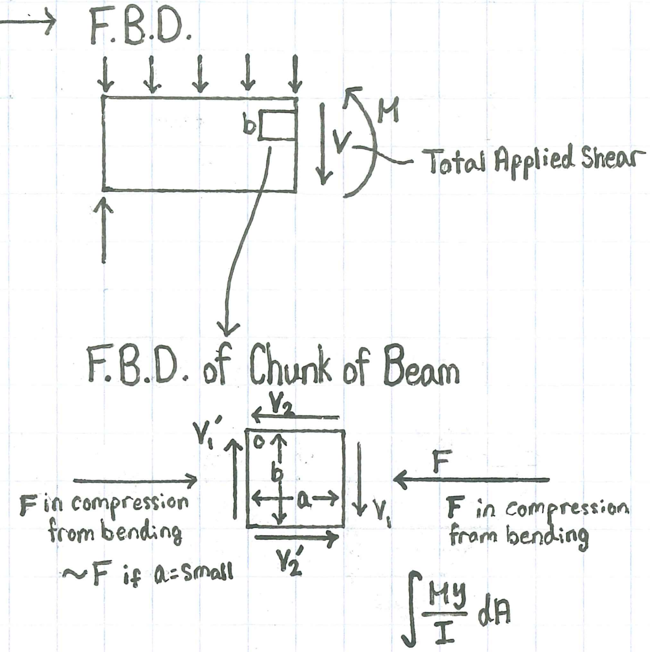
CIV102 - STRUCTURES and MATERIALS

Topic: Shear Stress

1)





V_1 = Part of Total Shear V

$\Sigma F_y = 0 \Rightarrow V_1' = V_1$

$\Sigma F_x = 0 \Rightarrow V_2 = V_2'$

And couple from V_2 will cancel out Force Couple from V_1

$\Sigma M_0 = 0$

$0 = -V_1 \cdot a + V_2 \cdot b$

$V_1 \cdot a = V_2 \cdot b$

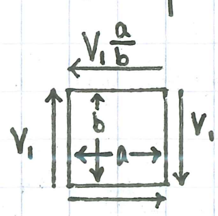
$V_2 = V_1 \frac{a}{b}$

2) Define Shear Stress

τ = Shear Stress = $\frac{\text{Force}}{\text{Area}}$ = MPa

Force is parallel to Area

t = thickness

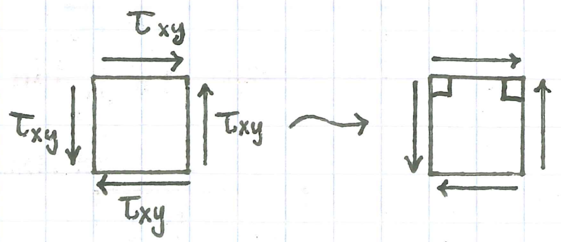


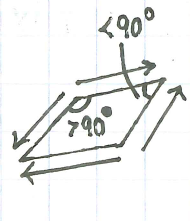
$\tau_{\text{on sides}} = \frac{\text{Force}}{\text{Area}} = \frac{V_1}{b \cdot t}$

$\tau_{\text{on top}} = \frac{V_1 a}{b a t} = \frac{V_1}{bt}$

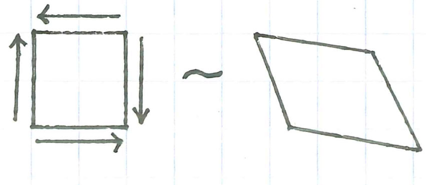
Shear stress on top + bottom + left + right sides are all the same

$\tau_{xy} = V_{xy}$

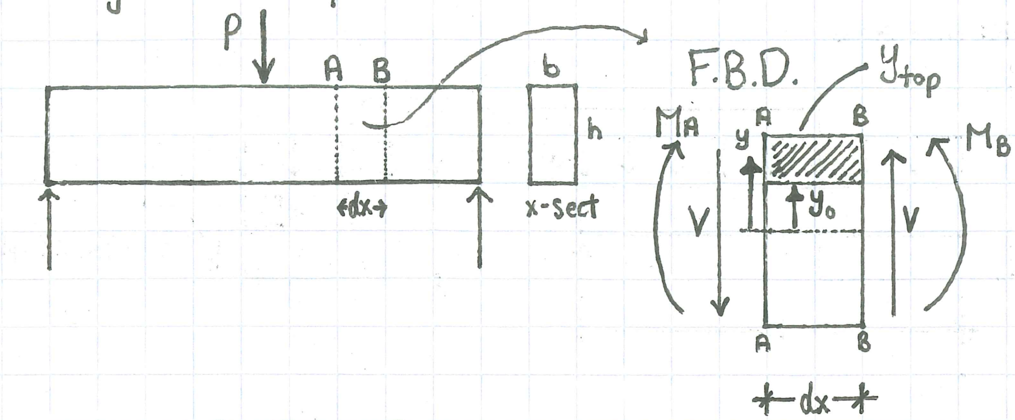




deformed shape



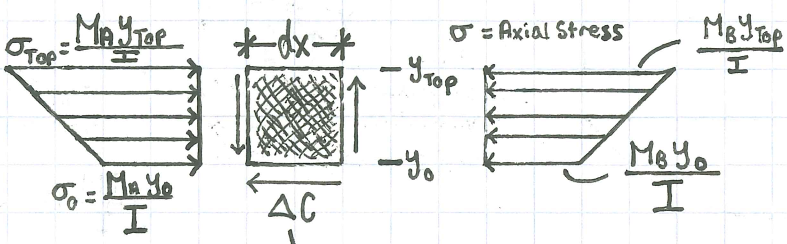
3) τ_{xy} Versus Depth



$$\Sigma M_A = 0$$
$$0 = M_B - M_A + V\cdot dx$$
$$M_A = M_B + V\cdot dx$$

$$\sigma_x = \frac{My}{I}$$

F.B.D. of Shaded Region



Caused by $M_A \neq M_B$
Induces τ_{xy} Stresses

Calculate ΔC

$$\text{Force on Left} = \int_{y_o}^{y_{top}} \sigma \, dA = \int_{y_o}^{y_{top}} \frac{M_A y}{I} \, dA$$

$$\text{Force on Right} = \int_{y_o}^{y_{top}} \frac{M_A y}{I} \, dA$$

$$\Delta C = \frac{(M_B - M_A)}{I} \int_{y_o}^{y_{top}} y \, dA$$

Value of $(M_B - M_A)$

$$(M_B - M_A) = M_B - (M_B + V\,dx)$$
$$= -V dx$$

Take absolute value

$$(M_B - M_A) = V dx$$

1855 Jourawski (Zurauski)

$$\tau_{xy} \text{ @ Depth } y_o = \frac{\Delta C}{dx \cdot b}$$

b: Width of cross section @ depth y_o

$$\tau_{xy} = \frac{V\,\cancel{dx}}{I} \int_{y_o}^{y_{top}} y\,dA \cdot \frac{1}{\cancel{dx}\cdot b}$$

$$\tau_{xy} = \frac{V}{Ib} \int_{y_o}^{y_{top}} y\,dA$$

$\int_{y_o}^{y_{top}} y\,dA$: 1st Moment of Area Q

$$\boxed{\tau_{xy} = \frac{VQ}{Ib}}$$