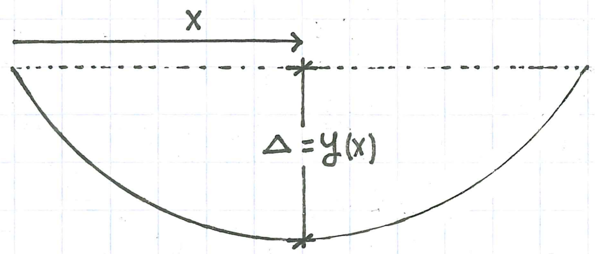
CIV102 - STRUCTURES and MATERIALS

Topic: MAT + Δ + θ

1)

$y(x)$	Δ	Displacement	$\int \theta \, dx$	Deformations
$\frac{dy}{dx}$	θ	Slope	$\int \frac{M}{EI} dx$	
$\frac{d^2y}{dx^2}$	ϕ	Curvature	$\frac{M}{EI}$	
$EI\frac{d^2y}{dx^2}$	M	Moment	$\int V \, dx$	* EI
$EI\frac{d^3y}{dx^3}$	V	Shear	$\int w \, dx$	Forces
$EI\frac{d^4y}{dx^4}$	w	Distributed Load		

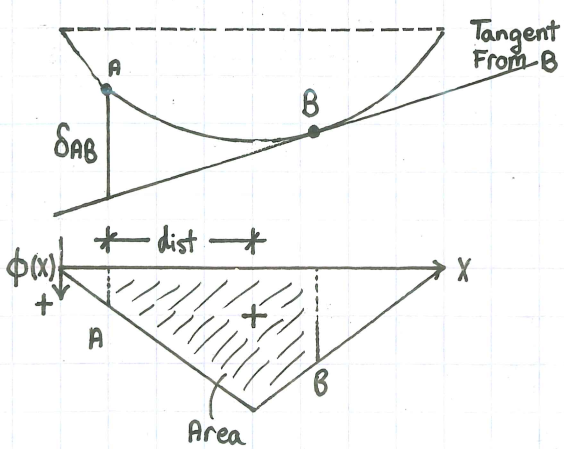


$$\Delta = \iiiint \frac{w}{EI} dx$$

2) Moment Area Theorems (MAT)

MAT #1 $\longrightarrow \Delta\theta_{AB} = |\theta_B - \theta_A| = \int_A^B \phi(x)\,dx$ = Area under ϕ diagram between A and B

MAT #2 $\longrightarrow \delta_{DT} = \underbrace{\overline{x}_{DT}}_{\text{Distance}} \underbrace{\int_D^T \phi(x)\,dx}_{\text{Area}}$ = Distance between the deflected beam at D and the line that is tangent to T equals the <u>area</u> under the ϕ diagram times the <u>distance</u> from the centroid of that diagram to D



δ_{AB} = Area · dist

A - Where is δ

B - Where is Tangent

A - Where is Distance

Note: $\delta_{AB} \neq \delta_{BA}$

Procedure to Find θ and Δ of Beams

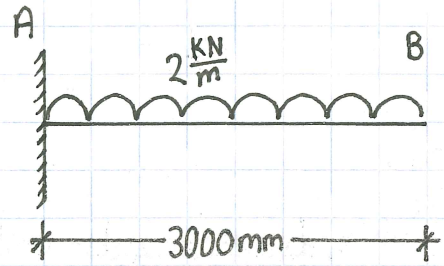
1) Calculate the reaction forces, SFD, BMD, ϕ Diagram
2) Sketch the displaced shape
   - ↳ The beam can't move up/down at support
   - ↳ The BMD tells you which way it is curving
3) Find places where the Δ or θ are known
   - ↳ Supports
   - ↳ Loading Conditions
4) Use MAT #1 to find θ
5) Use MAT #2 to find Δ

Senario 1 – Fixed Supports

At fixed support, $\theta = 0$

Example Find θ_B and ΔB

EI Constant $= 5 \times 10^{13}\ Nmm^2$



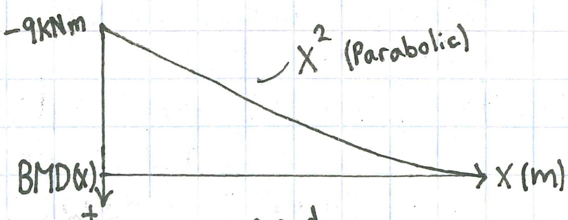
$\Delta\theta_{AB} = |\theta_B - \cancel{\theta_A}^{\,0}|$

θ_B = Area under ϕ

$= \frac{1}{3} bh$

$= \left(\frac{1}{3}\right)(3000mm)(180.0 \times 10^{-9}\ \frac{rad}{mm})$

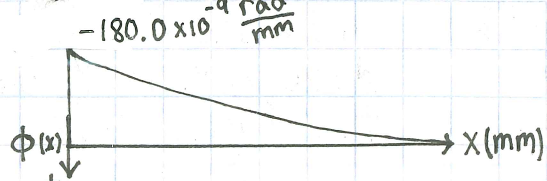
$\boxed{\theta_B = 180.0 \times 10^{-6}\ rad}$



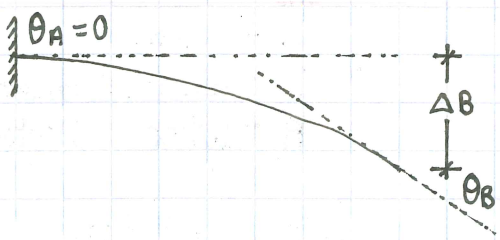
$\delta_{BA} = \Delta_B = (180.0 \times 10^{-6})\left(\frac{3}{4}\right)(3000mm)$

$\boxed{\Delta B = 0.405mm}$

$\phi = \frac{M}{EI}$



Displaced Shape (NTS)

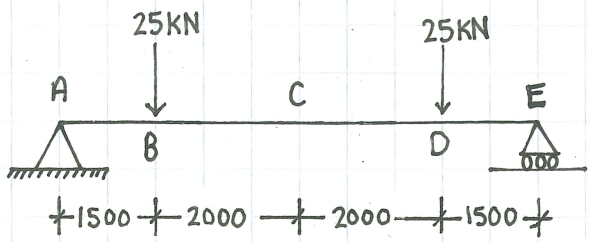


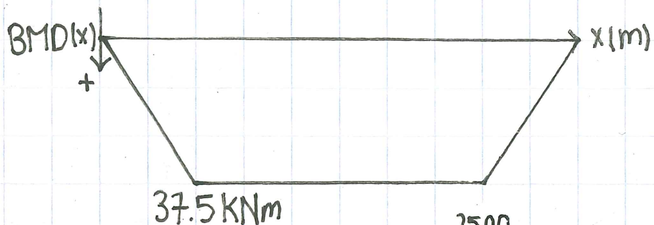
Senario 2 - Symmetry

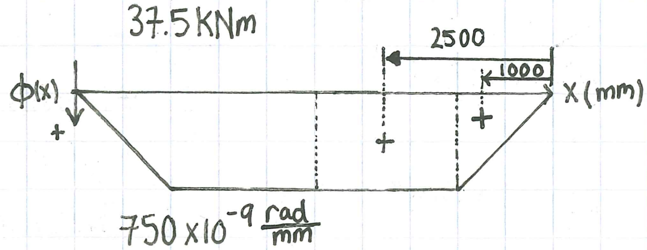
If the geometry and loading are symmetrical, then max deflection location is known

↳ At the location of max Δ, $\theta = 0$

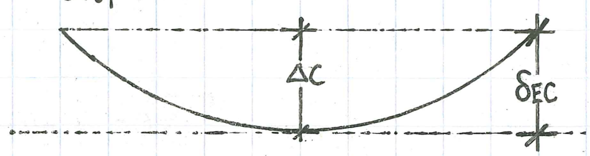
Example Find Δ_C $EI = 5\times10^{13}\ Nmm^2$







Displaced Shape



$\theta_c = 0$

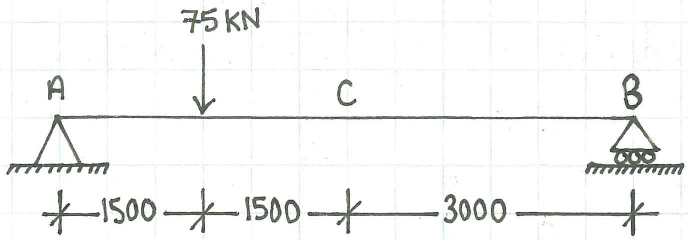
$$\delta_{EC} = \Delta_C = (2000mm)(750\times10^{-9}\tfrac{rad}{mm})(2500mm) + (\tfrac{1}{2})(1500mm)(750\times10^{-9}\tfrac{rad}{mm})(1000mm)$$

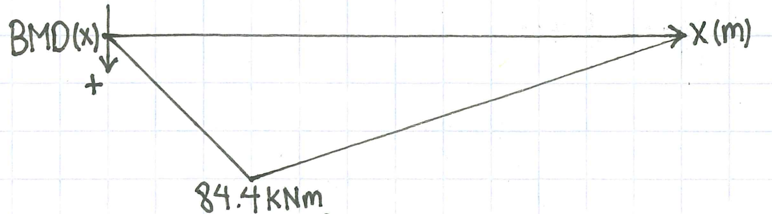
$$\boxed{\Delta_C = 4.31mm}$$

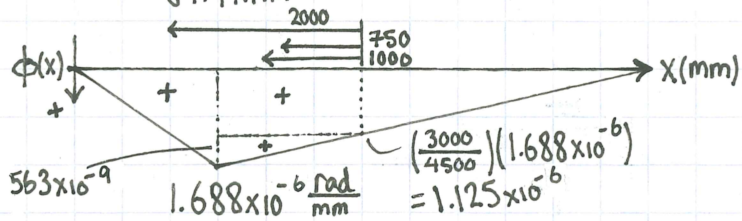
Scenario 3 – Everything Else

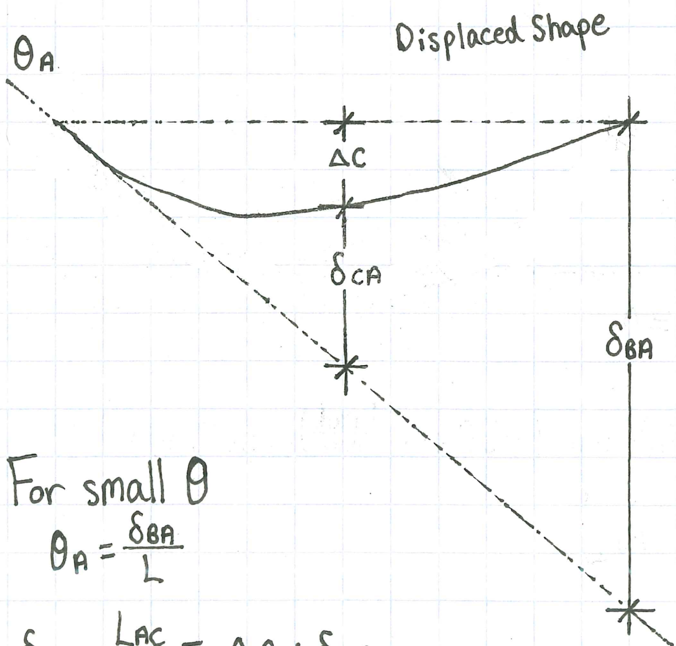
No horizontal tangents are known. Use similar triangles.

Example: Find Δ_C $\quad EI = 5\times10^{13}\ \text{Nmm}^2$









For small θ

$$\theta_A = \frac{\delta_{BA}}{L}$$

$$\delta_{BA}\cdot\frac{L_{AC}}{L_{AB}} = \Delta C + \delta_{CA}$$

$$\tfrac{1}{2}\delta_{BA} = \Delta C + \delta_{CA}$$

$$\delta_{BA} = 17.72\,\text{mm}$$

$$\delta_{CA} = (1500)(1.688\times10^{-6})(\tfrac{1}{2})(2000) + (1500)(1.125\times10^{-6})(750) + (1500)(563\times10^{-9})(\tfrac{1}{2})(1000)$$

$$\delta_{CA} = 4.22\,\text{mm}$$

$$\Delta C = \tfrac{1}{2}\delta_{BA} - \delta_{CA} = (\tfrac{1}{2})(17.72) - 4.22$$

$$\boxed{\Delta C = 4.64\,\text{mm}}$$