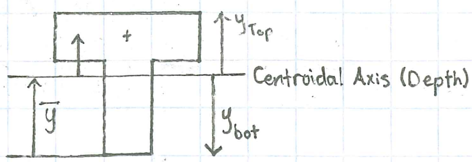
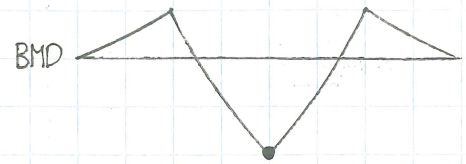
CIV102 - STRUCTURES and MATERIALS

Topic: $\theta + \Delta$

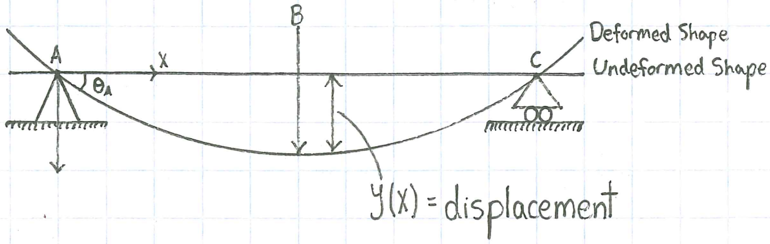
1) $\sigma = \frac{My}{I}$





$$|y_{bot}| > |y_{top}|$$

2) Displacements of Beams



Small Displacements

$$\theta_A = \text{Angle} = \text{Shape} = \frac{dy}{dx} = \tan\theta$$

$$\phi \longrightarrow M = EI\phi$$

$$\phi = \frac{M}{EI}$$

$$\phi = \frac{d\theta}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

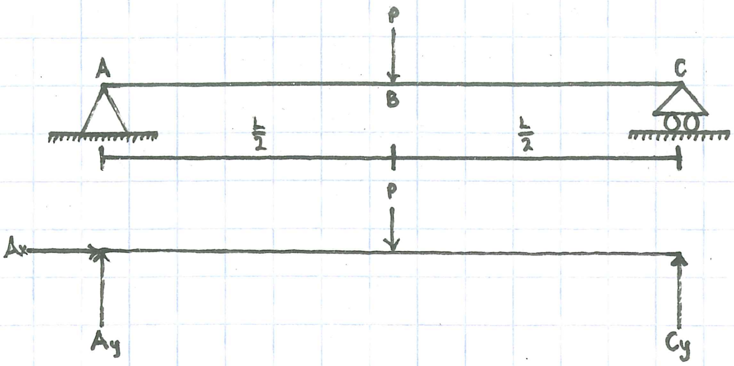
$$\boxed{\phi = \frac{d^2y}{dx^2}}$$ if angles are small

For larger angles

$$\phi = \frac{\frac{d^2y}{dx^2}}{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}$$

3) ϕ diagram

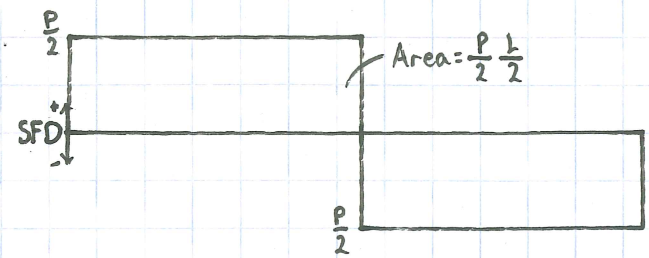
$$\phi = \frac{M}{EI}$$

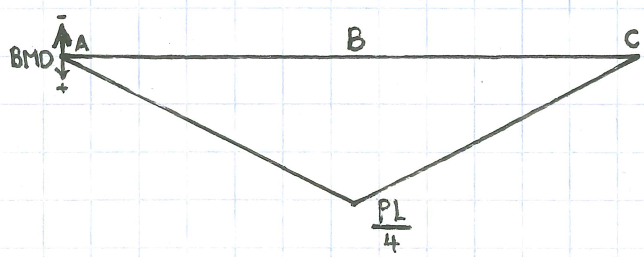


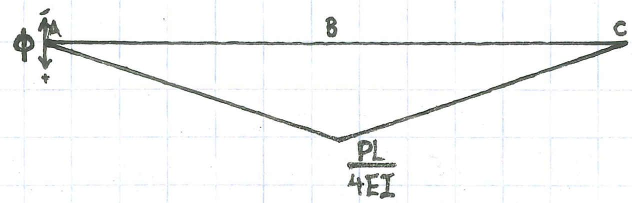
$$\Sigma F_x = 0,\ A_x = 0$$

$$\Sigma M_A = 0 = -P \cdot \frac{L}{2} + C_y \cdot L,\ C_y = \frac{P}{2}$$

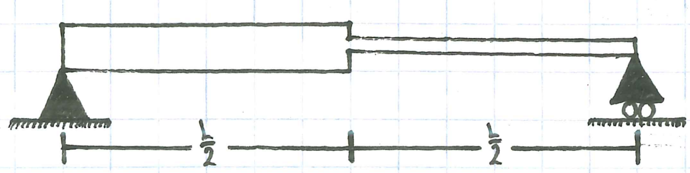
$$\Sigma F_y = 0 = A_y + C_y - P,\ A_y = \frac{P}{2}$$

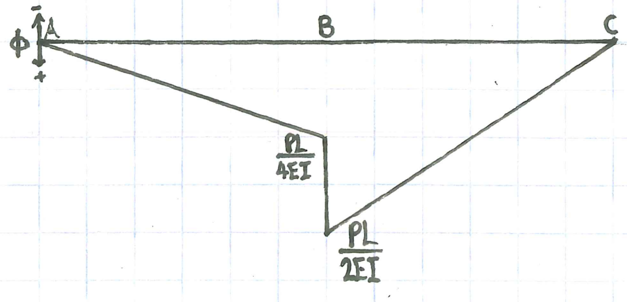






Problem 2





4) Solve for Displacements and Angles

- Calculus
- Moment Area Theorems

A) MAT#1 $\longrightarrow \theta$

$\phi = \frac{d^2y}{dx^2}$, Integrate Once

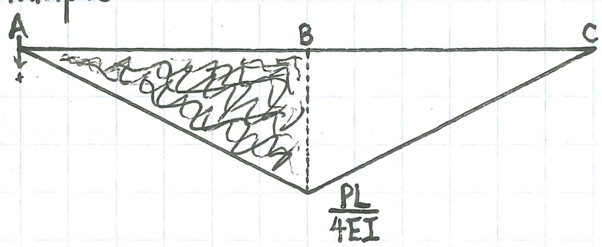
$$\int \phi \, dx + C = \frac{dy}{dx} = \theta$$

$$\theta = \int \phi \, dx + C$$

Moment Area Theorem #1

The change in slope between A and B in a beam is equal to the area under ϕ diagram between A and B.

Example



$\theta_A = ?$

by symmetry $\theta_B = \text{Zero}$

$$\theta_A - \theta_B = \text{Shaded Area} = \frac{L}{2} \cdot \frac{PL}{4EI} \cdot \frac{1}{2}$$

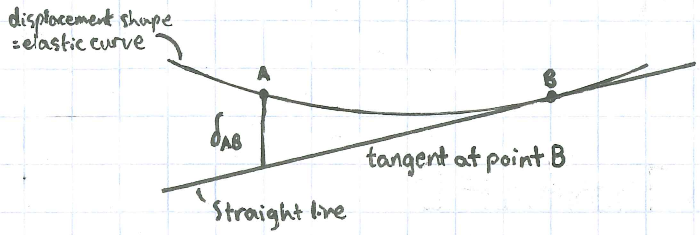
$$\boxed{\theta_A = \frac{PL^2}{16EI}}$$

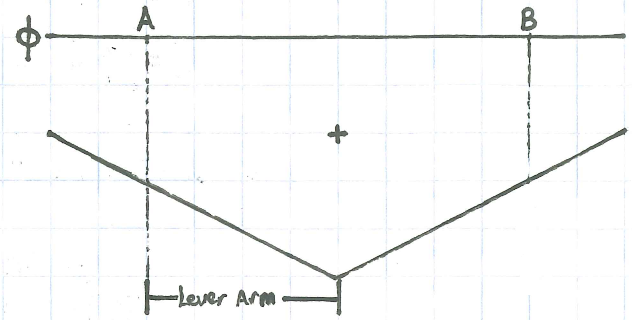
B) MAT # 2

$$\int \phi \, dx + C_1 = \frac{dy}{dx}$$

$$\underbrace{\iint \phi \, dx}_{\text{First Moment of Area}} + \underbrace{C_1 x + C_2}_{\text{Equation of a Line}} = y$$

displacement perpendicular to undeformed shape from the straight line





Moment Area Theorem #2

The tangental deviation δ_{AB} at point A between the elastic curve and the tangent drawn from B is equal to the first moment of area of the Φ diagram between A and B with moments taken about point A.

δ_{AB}

A → Where to get δ
B → Where is the tangent
A → Where to get lever arm

$\delta_{AB} = A_B \cdot (\text{lever arm})$