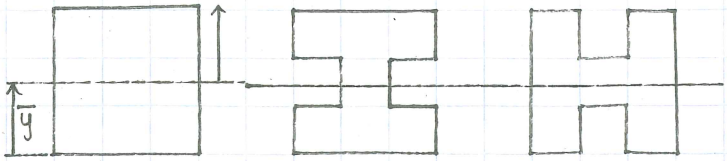
CIV102 - STRUCTURES and MATERIALS

Topic: $\bar{y}$ and I

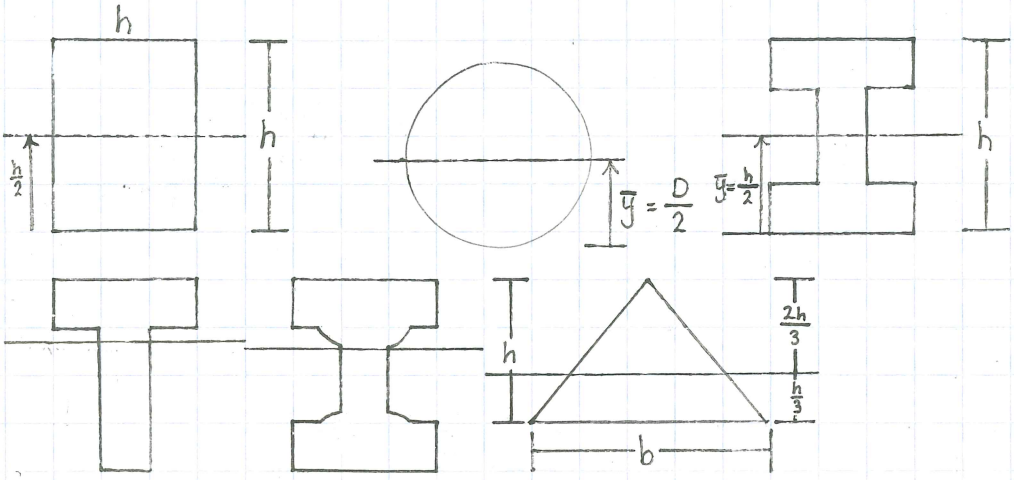
$\sigma = \frac{My}{I}$



1) Why I?

$\sigma = \frac{My}{I}$ $\quad M = EI\phi$

2) Centroids

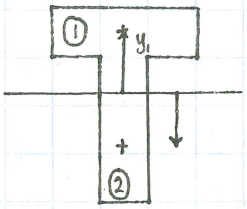


Last Class

$N = E\phi \int y\,dA$

If only M, N = 0

$0 = \int y\,dA$

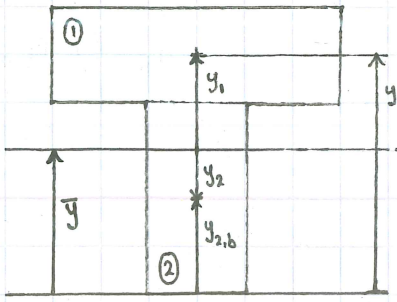


$0 = A_1 y_1 + A_2 y_2$

y_1 = distance from centroid of A_1 to centroid of entire section

A_1, A_2 = Positive

y_1, y_2 = One Positive, One Negative



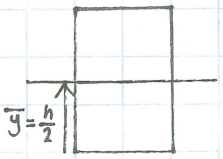
$$0 = A_1 \cdot y_1 + A_2 \cdot y_2$$
$$= A_1(y_{1,b} - \bar{y}) + A_2(y_{2,b} - \bar{y})$$
$$= \sum_{i=1}^{n} A_1(y_{1,b} - \bar{y})$$
$$0 = \sum_{i=1}^{n} A_i y_{i,b} - \sum_{i=1}^{n} A_i \cdot \bar{y}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i \cdot y_{i,b}}{\sum_{i=1}^{n} A_i} = \frac{\sum_{i=1}^{n} A_i \cdot y_{i,b}}{\text{Total Area}}$$

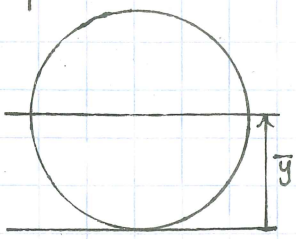
3) Calculate I

$$I = \int y^2 \, dA$$

I for some basic shapes

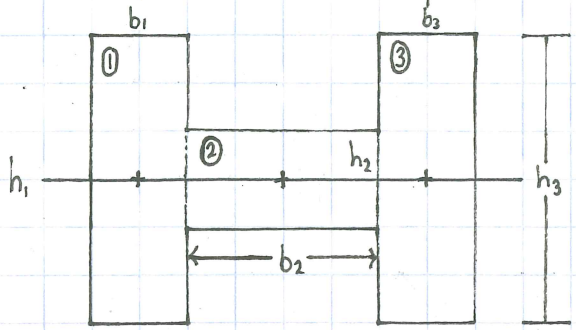


$$I = \frac{bh^3}{12}$$

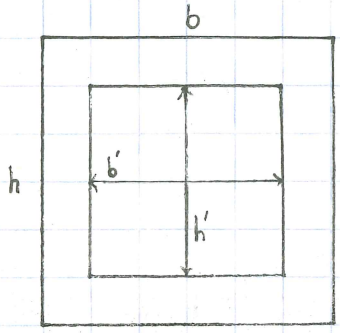


$$I = \frac{\pi D^4}{64}$$

a) Cases where all subparts have same centroidal axis height

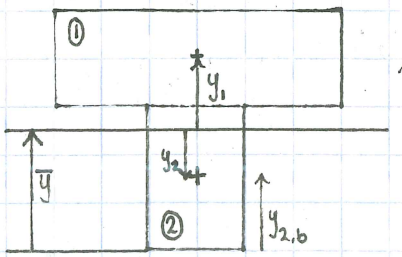


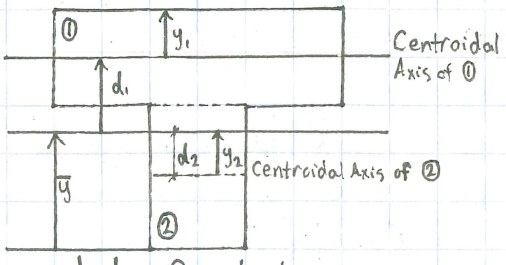
$$I_{total} = I_1 + I_2 + I_3$$
$$= \frac{b_1 h_1^3}{12} + \frac{b_2 h_2^3}{12} + \frac{b_3 h_3^3}{12}$$



$$I_{total} = I_{Solid} - I_{Hollow}$$
$$= \frac{bh^3}{12} - \frac{b'h'^3}{12}$$

b) I if centroidal axis do not align





d_1, d_2 = Constants

d_1 = distance from centroid of A_1 to Centroid of whole section

$$I = \int y^2 \, dA$$
$$= \int_{A_1} (y_1 + d_1)^2 \, dA + \int_{A_2} (y_2 + d_2)^2 \, dA$$
$$= \int_{A_1} y_1^2 \, dA + \int_{A_1} 2 y_1 d_1 \, dA + \int_{A_1} d^2 \, dA + \int y_2^2 \, dA + \int 2 y_2 d_2 \, dA + \int d_2^2 \, dA$$
$$= I_o + 2d_1 \cancel{\int y_1 \, dA}^{0} + d_1^2 \int dA + I_{o2} + 2d_2 \cancel{\int y_2 \, dA}^{0} + d_2^2 \int dA$$

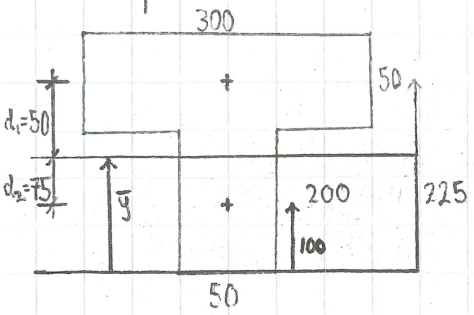
I_o: I of area 1 about its own centroid $= \frac{bh^3}{12}$

$\int y \, dA = 0$ as

$$\therefore I = I_{o1} + I_{o2} + A_1 d_1^2 + A_2 d_2^2$$

Parallel Axis Theorem

Example



$$\bar{y} = \frac{A_1 y_{1,b} + A_2 \cdot y_{2,b}}{A_{Total}}$$

$$= \frac{300 \cdot 50 \cdot 225 + 200 \cdot 50 \cdot 100}{300 \cdot 50 + 200 \cdot 50}$$

$$= 175\,mm$$

$$I = \frac{b_1 h_1^3}{12} + \frac{b_2 h_2^3}{12} + A_1 d_1^2 + A_2 d_2^2$$

$$\frac{b_1 h_1^3}{12} = 3.125 \times 10^6\,mm^4$$

$$\frac{b_2 h_2^3}{12} = 33.3 \times 10^6\,mm^4$$

$$A_1 d_1^2 = 37.5 \times 10^6\,mm^4$$

$$A_2 d_2^2 = 56.25 \times 10^6\,mm^4$$

$$\sum I_{Total} = 130.2 \times 10^6\,mm^4$$