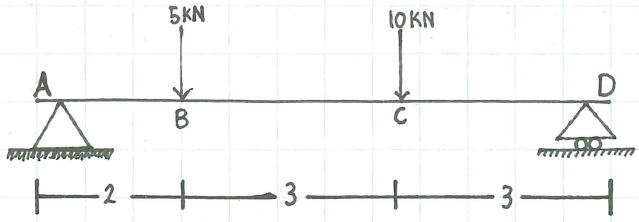
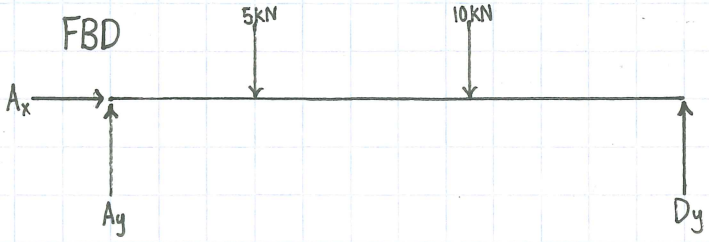
CIV102 - STRUCTURES and MATERIALS

V is slope of BMD
Change in Moment = Area Under SFD

1)





$\Sigma F_x = 0$, $A_x = 0$

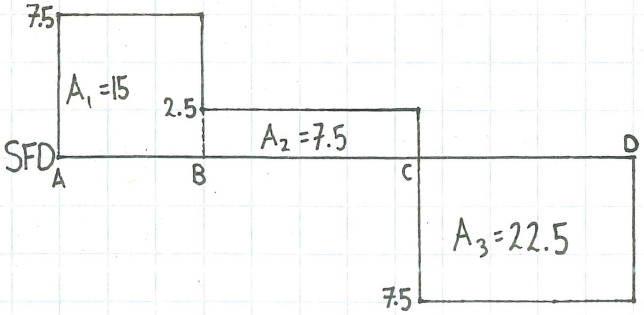
$\Sigma M_A = 0 = -5 \cdot 2 - 10 \cdot 5 + D_y \cdot 8$

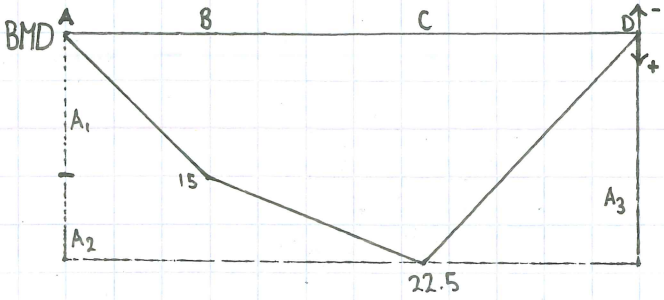
$D_y = 7.5\text{kN}$

$\Sigma F_y = 0$

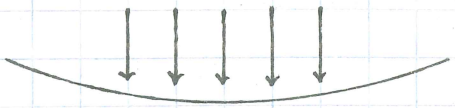
$0 = -5 - 10 + A_y + D_y$

$A_y = 7.5\text{kN}$

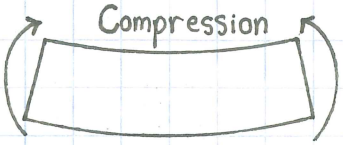




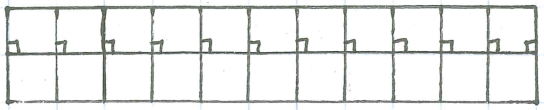
Draw on tension side

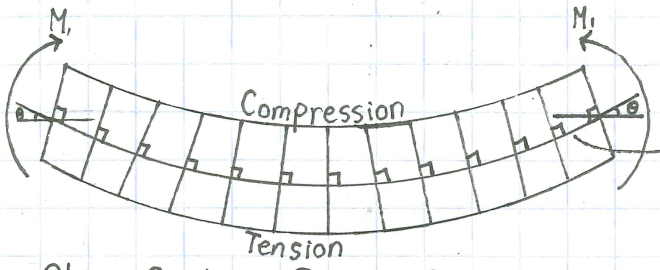


= Positive Bending
Positive Moments



2) Calculate Bending Stresses





$\varepsilon = 0$ at Neutral Axis = Centroid of X-Section

Plane Sections Remain Plane

$$\phi = \frac{d\theta}{dx} = \text{Curvature}$$

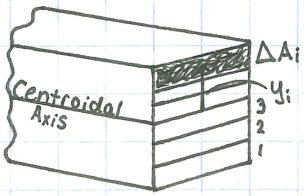
$$= \frac{\theta_2 - \theta_1}{L}$$

$$\varepsilon(y) = \phi y$$

$$M = EI\phi$$

$$I = \int y^2 dA = \frac{bh^3}{12}$$

3) $M \longleftrightarrow \sigma$



Strain in layer i

$$\varepsilon = \phi \cdot y \cdot i$$
$$\sigma = \varepsilon \cdot E$$
$$\sigma = \phi E y_i$$

Force in layer i

$$\Delta F_i = \phi E y_i \Delta A_i$$

Axial total force $= \sum_{i=1}^{n} \phi E y_i \Delta A_i$

Take limit as $\Delta A_i \to 0$ $\boxed{N = \phi E \int y\, dA}$

We want moment effects only

$$0 = E\phi \int y\, dA$$

$$\boxed{0 = \int y\, dA} \to \underline{\text{Use tomorrow}}$$

$$\text{Moment} = \sum_{i=1}^{n} \Delta F_i \cdot \text{lever arm}$$

$$= \sum_{i=1}^{n} \Delta F_i \cdot y_i$$

$$M = \sum_{i=1}^{n} \overbrace{\phi E y_i \cdot \Delta A_i}^{\Delta F_i} \cdot y_i$$

$$M = \phi E \sum_{i=1}^{n} y_i^2 \cdot \Delta A_i$$

Take $\lim \Delta A_i \to 0$

$$M = E\phi \int y^2 \, dA$$

$$M = EI\phi$$

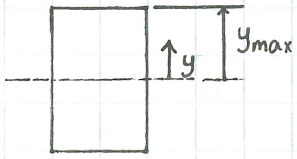
Above we said $\sigma = \phi E y$

$$\phi = \frac{\sigma}{Ey}$$

Sub in

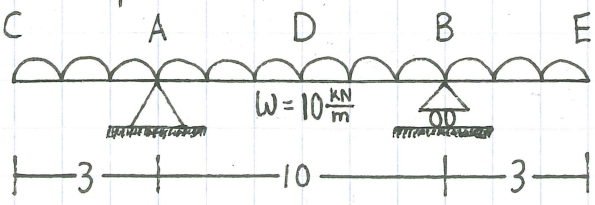
$$M = EI \cdot \frac{\sigma}{Ey}$$

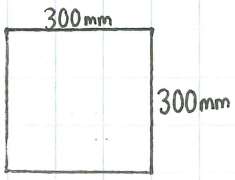
$$\boxed{\sigma = \frac{My}{I}}$$ Navier's Equation



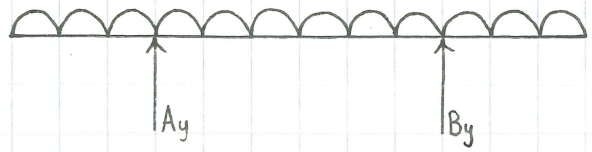
$$\boxed{\sigma_{max} = \frac{M_{max} y_{max}}{I}}$$

4) Example



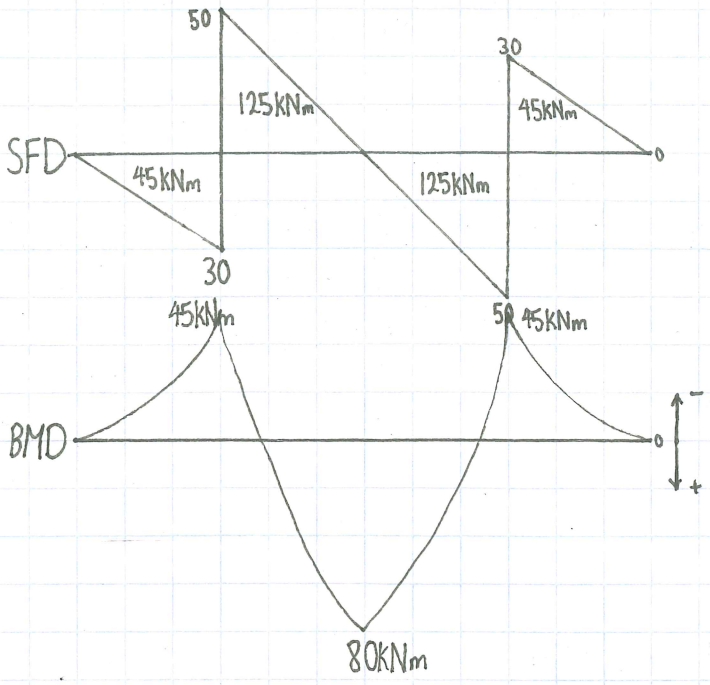


$\sigma_{max} = ?$



Total Force Down = 10·16 = 160kN

Symmetry $A_y = B_y = 80\text{kN}$



$$\sigma = \frac{My}{I} \qquad I = \frac{bh^3}{12} = 675 \times 10^6 \, mm^4$$

$$\sigma_{max} = \frac{M_{max} \cdot y_{max}}{I} \qquad y_{max} = 150 = \frac{h}{2}$$

$$= \frac{80 \times 10^6 \, Nmm \cdot 150 \, mm}{675 \times 10^6 \, mm^4} \qquad M_{max} = 80 kNm$$

$$= 17.78 MPa$$