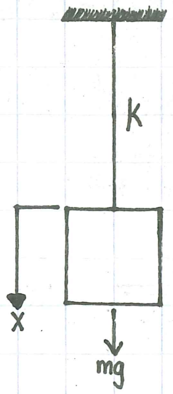
CIV102 - STRUCTURES and MATERIALS

Topic: Dynamic Amplification Factor (DAF) and Resonance

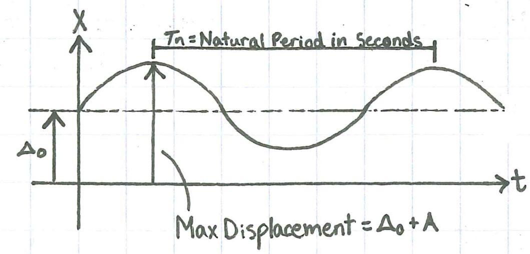
1) Case With No Damping



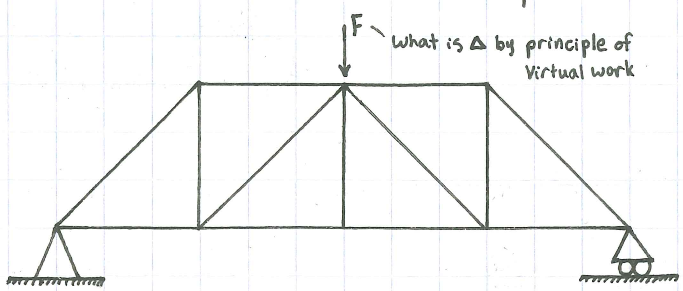
$$m\frac{d^2X}{dt^2} + kX = mg$$

$$X(t) = A\sin(\omega_n t + \phi) + \Delta_o$$

(ω_n — natural frequency in $\frac{\text{radians}}{\text{second}}$)



Static Displacement $= \Delta_o = \frac{mg}{k}$



For 1 Point Load $\rightarrow f_n \simeq \frac{15.76}{\sqrt{\Delta_o}}$ if Δ_o in mm

For Members With Uniform Loads $\rightarrow f_n = \frac{17.76}{\sqrt{\Delta_o}}$

2) Vibration With Damping

Damping means vibrational energy is statically absorbed

- Concept called critical damping $\longrightarrow$ PHY293
- We define damping as β = fractional critical damping $\begin{cases} 0.01 = \beta \\ 0.05 = \beta \end{cases}$

Differential Equation

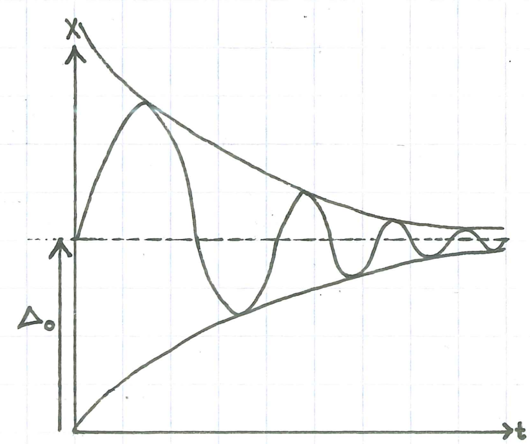
$$m\frac{d^2X}{dt^2} + \underbrace{C\frac{dX}{dt}}_{\text{damping}} + kX = mg$$

$C = 2\beta\sqrt{m \cdot k}$

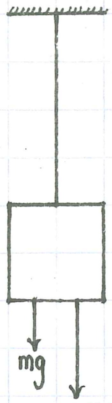
$$X(t) = \underbrace{Ae^{-\beta \cdot \omega_n \cdot t}}_{\text{Exponential Decay}} \sin(\omega_n \underbrace{\sqrt{1-\beta^2}}_{} t + \phi) + \Delta_o$$

β = Small

β^2 = Really Small



3) Forced Vibration with Damping



ω, f = forcing frequency
ω_n, f_n = natural frequency

$F(t) = F_o \sin(\omega \cdot t)$

Differential Equation

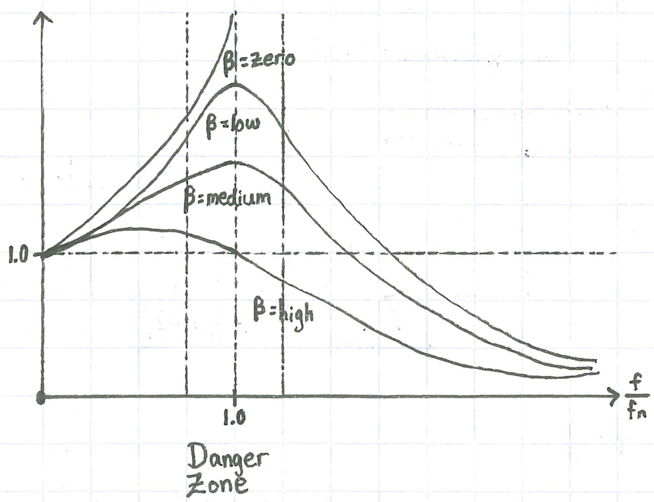
$$m\frac{d^2x}{dt^2} + 2\beta\sqrt{mk}\frac{dx}{dt} + kx = F_o \sin(\omega t) + mg$$

$$X(t) = DAF \cdot \frac{F_o}{K} \cdot \sin(\omega t + \phi) + \Delta_o$$

DAF: dynamic amplification factor; ω: forcing frequency; Δ_o: static displacement

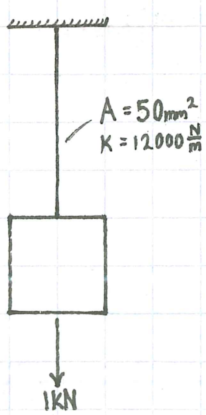
$$DAF = \frac{1}{\sqrt{\left(1-\left(\frac{f}{f_n}\right)^2\right)^2 + \left(\frac{2\beta f}{f_n}\right)^2}} \quad \text{(unitless)}$$

Plot DAF



Maximum force experienced by structure
Apparent $F_{max} = DAF \cdot F_o + mg$

4) Example



Statics

$\sigma = \frac{F}{A} = \frac{1000N}{50mm^2} = 20\,MPa$

$\Delta = \frac{F}{K} = \frac{1000N}{12000\frac{N}{m}} = 0.083m = 83.3mm$

$f_n = \frac{15.76}{\sqrt{\Delta}} = 1.727\,Hz$

Dynamics

Tells You → Dynamic Loading = 1000N ± 50N @ 2Hz

DAF = 2.78

$F_{max} = 1000N + DAF \cdot F_o$
$= 1000N + 2.78 \cdot 50$
$= 1139N \rightarrow \sigma = 22.8\,MPa$