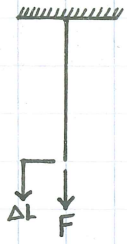
CIV102 - STRUCTURES and MATERIALS

Topic: Δ

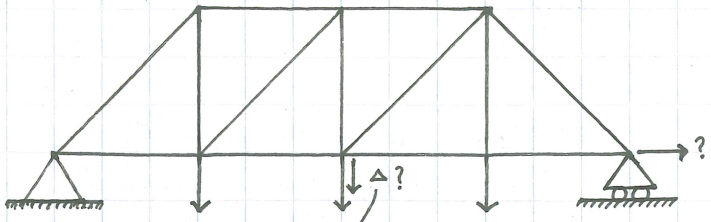
1) Displacements



$F = K \cdot \Delta L$

$F = \frac{AE}{L}(\Delta L)$

$$\Delta L = \frac{FL}{EA}$$



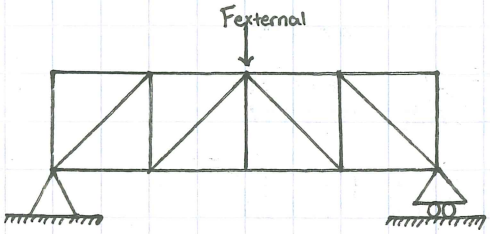
What is $\Delta_{allowable}$

$\Delta < \frac{L}{300} = \underline{OK}$

from live load only

We will use Energy!

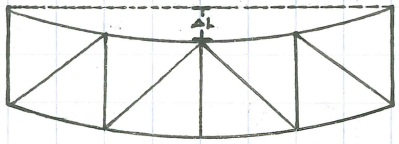
- Conservation of Energy



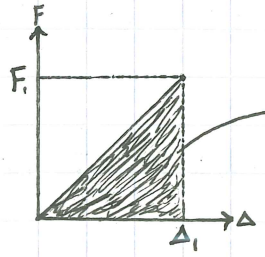
$W_{external}$ = Work done by external forces

$W_{internal}$ = Work absorbed by strain energy internally

$$\underline{W_{external} = W_{internal}}$$



Consider External Force/Work



Area Under Curve = Energy

$W_{ext} = \int F \, d\Delta$

$Area = \frac{F_1 \Delta_1}{2}$ ← Goal

$F_{external}$

Internal Forces + Work

For each element, i

$$\text{Strain Energy} = \frac{F_i \cdot \Delta_i}{2}$$

(F_i: Internal Force; Δ_i: change in length of element i)

$$\Delta_i = \frac{F_i L_i}{E_i A_i}$$

$$\text{Energy} = \frac{F_i \cdot F_i L_i}{E_i A_i 2}$$

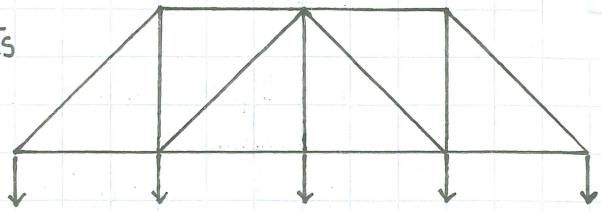
$W_{external} = W_{internal}$

$$\frac{1}{2} F_{external} \cdot \Delta_i = \sum_{i=1}^{n} \frac{F_i F_i L_i}{2 E_i A_i}$$

$F_{external}$: Known; Δ_i: goal; $\sum$: All known

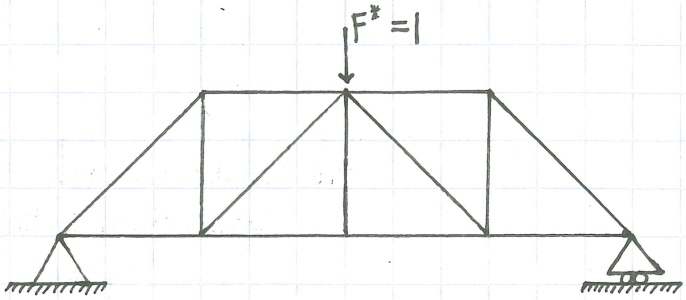
2) Virtual Work

- Real system will have actual displacements
- External Work/Force
- Internal Work/Force



Virtual System = Same Structure
- None of original loads included
- Apply a Force F^* at location of interest in direction of interest

F^* = Virtual Force = 1N = Unit Load



How to Apply Principle of Virtual Work (PVW)

1) For Real System = Real Applied Loads
- Solve for internal forces
- Solve for internal change in member lengths $\boxed{\Delta_i = \frac{F_i \cdot L_i}{E_i \cdot A_i}}$

2) Virtual System
- Remove real loads + Apply $F^* = 1N$
- Solve for internal Forces

Apply PVW

External VW = Internal VW

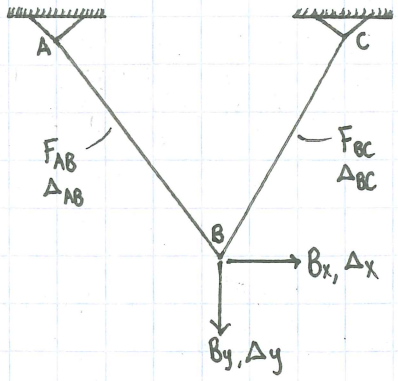
$F^* \cdot \Delta_{ext} = \sum F_i^* \cdot \Delta_i$ ($\frac{1}{2}$ factors are missing for virtual work)

Virtual — Real

$$F^* \cdot \Delta_{ext} = \sum F_i^* \cdot \frac{F_i L_i}{E_i A_i}$$

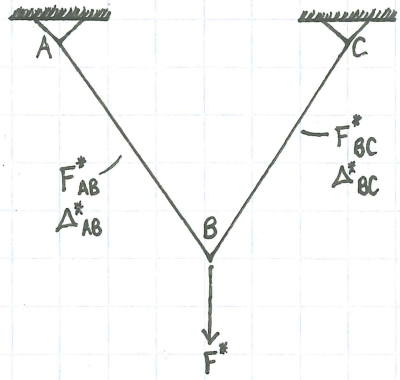
Why does it work?

Real



$$\frac{B_x \cdot \Delta_x}{2} + \frac{B_y \cdot \Delta_y}{2} = \frac{F_{AB} \cdot \Delta_{AB}}{2} + \frac{F_{BC} \cdot \Delta_{BC}}{2} \quad (1)$$

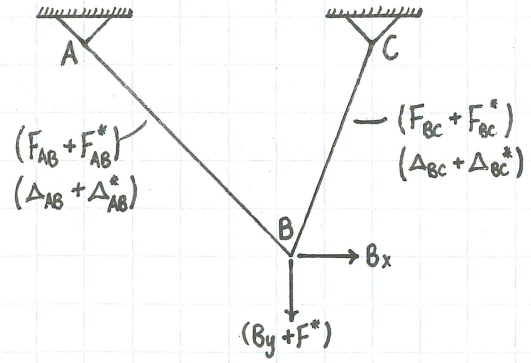
Virtual



$$\frac{F^* \cdot \Delta^*_y}{2} + \frac{B^*_x \cdot \Delta^*_x}{2} = \frac{F^*_{AB} \cdot \Delta^*_{AB}}{2} + \frac{F^*_{BC} \Delta^*_{BC}}{2} \quad (2)$$

$B^*_x = 0$

Combine Real + Virtual



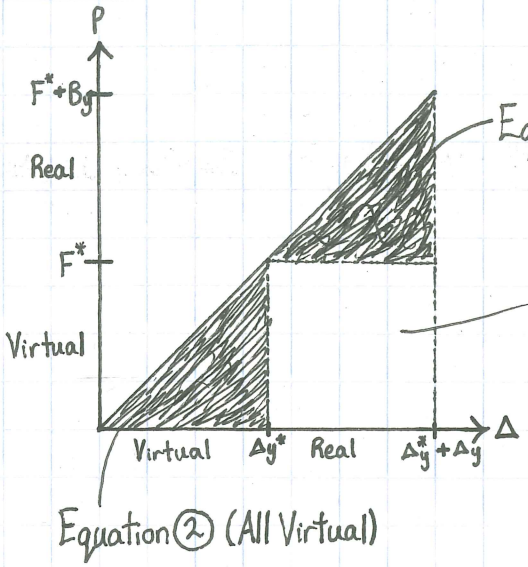
External = Internal

(3)

$$\frac{(B_x)(\Delta_x + \Delta_x^*)}{2} + \frac{(B_y + F^*)(\Delta_y + \Delta_y^*)}{2} = \frac{(F_{AB} + F^*_{AB})(\Delta_{AB} + \Delta^*_{AB})}{2} + \frac{(F_{BC} + F^*_{BC})(\Delta_{BC} + \Delta^*_{BC})}{2}$$

③ - ② - ①

Do it geographically



Equation ① (All Real)

Virtual Work = (Real Δ) · (Virtual Force)

Note: Not divided by 2

Equation ② (All Virtual)