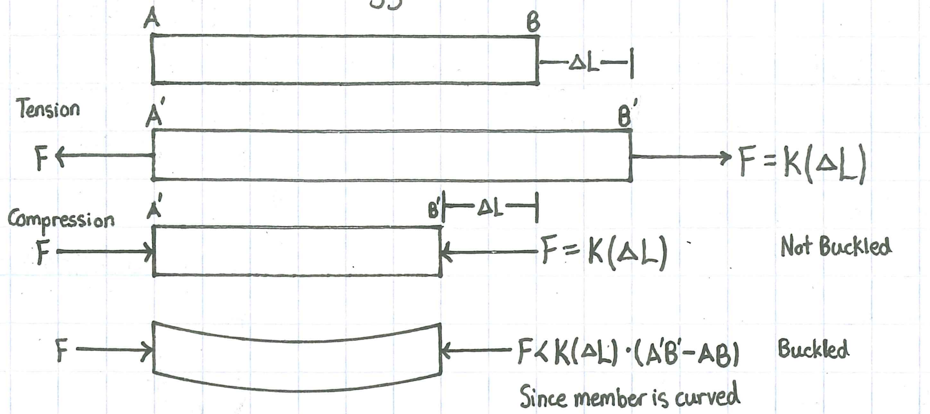
CIV102 – STRUCTURES and MATERIALS

Topic: Buckling

1) Why do things buckle?

- Achieve a lower energy state than if it had not buckled

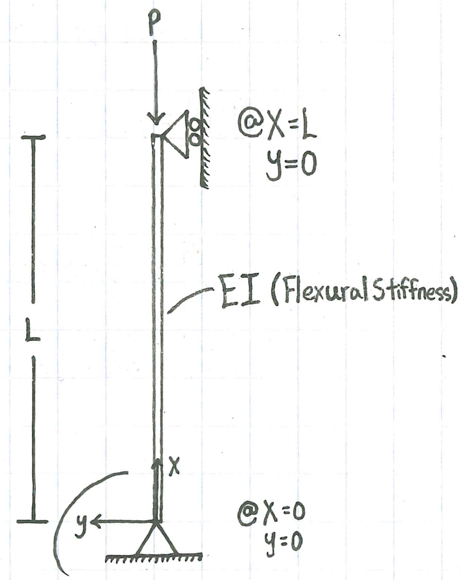


Euler 1757

$$P_E = P_{cr} = P_{critical}$$

(E = euler)

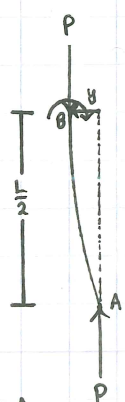
2) Derivation



Undeformed Shape

Assumptions

- EI = Constant Along Length ($M = EI \cdot \phi$)
- Ability to rotate @ top and bottom Column (Pin or Roller)
- Perfectly straight
- Small Displacements



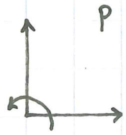
$$\Sigma M_B = 0$$

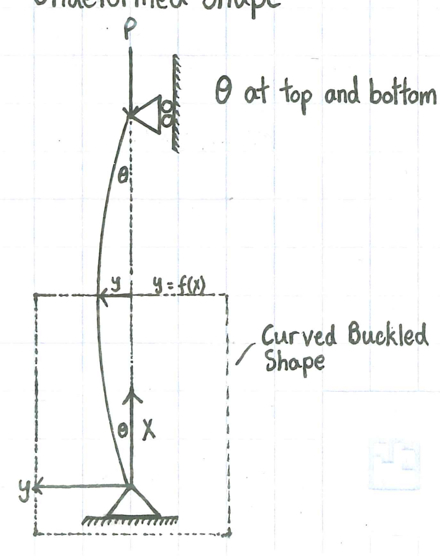
$$0 = -M + P \cdot y$$

$$M = P \cdot y \text{ @ Midheight}$$

In general:

$$M(X) = P \cdot y(X)$$





$$\phi = \text{Curvature} = -\frac{d\theta}{dx} = \frac{-d}{dx}\left(\frac{dy}{dx}\right) = -\frac{d^2y}{dx^2}$$

$$M = P \cdot y(X), \quad M = EI\phi$$

$$P \cdot y(X) = EI\phi$$

Differential Equation: $\boxed{P \cdot y = EI \frac{d^2y}{dx^2}}$

Guess Answer

$$y(x) = A\sin(\omega x + b)$$

$$P \cdot y = -EI\frac{d^2y}{dx^2}$$

$$y(x) = A\sin(\omega x + b)$$

$$\frac{dy}{dx} = A\omega\cos(\omega x + b)$$

$$\frac{d^2y}{dx^2} = -A\omega^2\sin(\omega x + b)$$

Substitute in

$$P \cdot A\sin(\omega x + b) = -EI(-A\omega^2\sin(\omega x + b))$$

$$P = EI\omega^2$$

$$\omega = \sqrt{\frac{P}{EI}}$$

Apply boundary conditions @ $x=0, y=0$

$y(0) = A\sin(\omega x + b) = 0$ — $A = 0$ (trivial case), $b = 0$

@ $x=L, y=0$

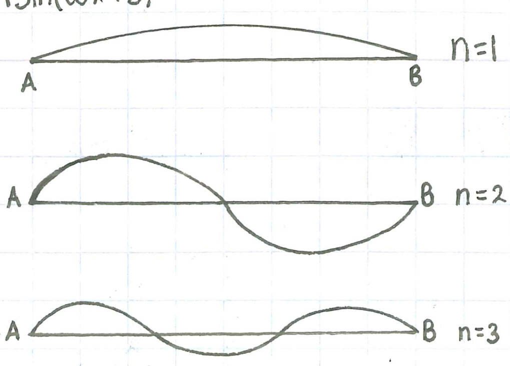
$$y(L) = A\sin(\omega L) = 0$$

$$\therefore \omega L = \pi, 2\pi, 3\pi, 4\pi, \cdots, n\pi$$

Safest Case = Practical Case $(n=1)$

$$P_{critical} = \frac{\pi^2 EI}{L^2}$$

$A\sin(\omega x + b)$



$$\omega = \sqrt{\frac{P}{EI}}, \quad \omega = \frac{n\pi}{L}$$

$$\frac{n\pi}{L} = \sqrt{\frac{P}{EI}}$$

$$\boxed{P_{cr} = \frac{\pi^2 EI n^2}{L^2}} \quad n = 1, 2, 3, 4, 5 \ldots$$

Euler buckling load