CIV102 - STRUCTURES and MATERIALS

Topic: Bending of Beams

1) Bending

$$\begin{cases} F = (\Delta L)K \\ \sigma = E\varepsilon \end{cases}$$

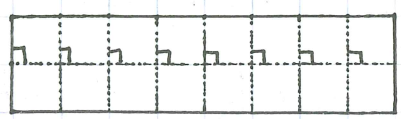
$M = \underline{\qquad} \cdot \underline{\qquad}$

relation betwixt moment and bentness

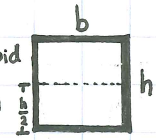
Measure of "bentness"

Assume
- Hooke's Law
- "Plane Sections Remain Plane" ⟶ Euler (Swiss), Bernoulli, Navier (French), Timochenko, Hooke's (English, 1678)

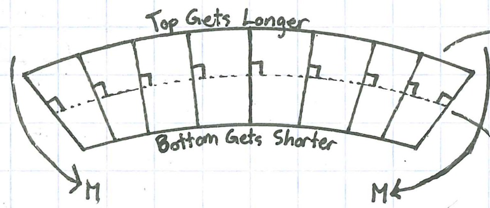
Undeformed Shape



Centroid Cross Section



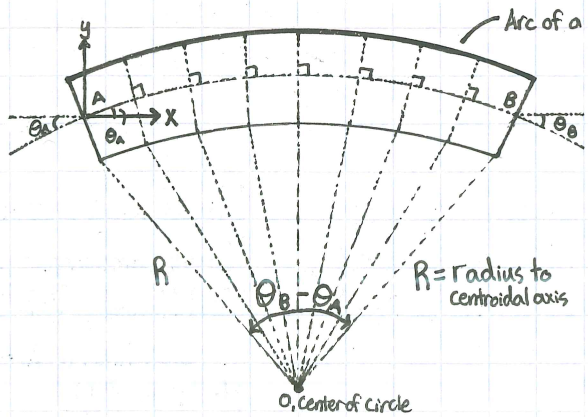
Deformed Shape



arc of a circle

length at centroidal axis does not change

2) Flexural Stiffness



Note: Diagram Not to Scale. Lines should be straight.

L_{AB} = Length Along Arc

$L_{AB} = (\theta_B - \theta_A) \cdot R$

From Hooke, we will assume this is constant.

If θ_A and θ_B are small, then the horizontal component of length L_{AB} will equal L_{AB}.

Define degree of "bentness" as curvature

$$\phi = \frac{(\theta_B - \theta_A)}{L_{AB}}$$

$$= \text{Rate of Change of Angle} = \text{Slope of Beam}$$

$$\phi = \frac{d\theta}{dx}$$

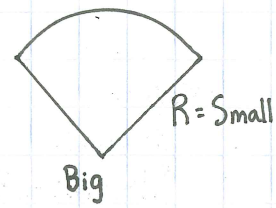
$$\phi = \underbrace{\frac{(\theta_B - \theta_A)}{L_{AB}}}_{\text{Whole Thing}} = \underbrace{\frac{(\theta_B - \theta_A)}{l_{AB}}}_{\text{Small Parts}}$$

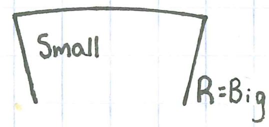
$$L_{AB} = (\theta_B - \theta_A) \cdot R$$

$$R = \frac{L_{AB}}{(\theta_B - \theta_A)}$$

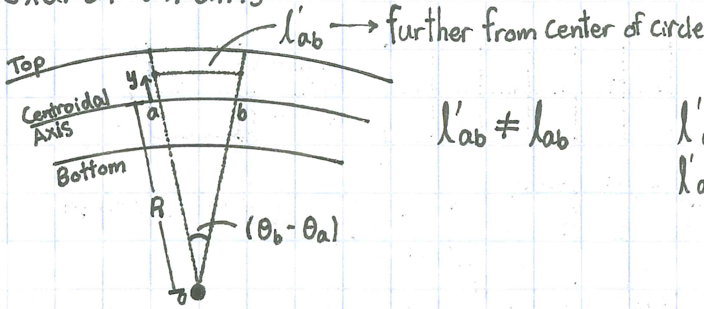
$$\phi = \frac{(\theta_B - \theta_A)}{L_{AB}}$$

$$\phi = \frac{1}{R}$$





3) Flexural Strains



$$l'_{ab} \neq l_{ab}$$

$$l'_{ab} = l_{ab}$$

$$l'_{ab} = (\theta_b - \theta_a) \cdot (R + y)$$

$$= (\theta_b - \theta_a) \cdot \left(\frac{1}{\phi} + y\right)$$

$$\phi = \frac{(\theta_b - \theta_a)}{l_{ab}} \longrightarrow (\theta_b - \theta_a) = \phi l_{ab}$$

$$l'_{ab} = \phi l_{ab} \left(\frac{1}{\phi} + y\right)$$

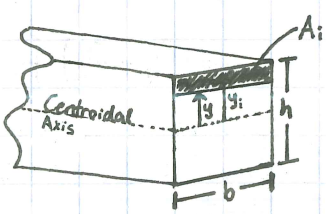
$$l'_{ab} = l_{ab} + \phi l_{ab} y$$

$$\varepsilon = \frac{\Delta L}{L}, \quad \Delta L = l'_{ab} - l_{ab}$$

$$\varepsilon = \frac{\phi l_{ab} \cdot y}{l_{ab}}$$

$$\boxed{\varepsilon = \phi \cdot y}$$

4) Relation of M to ϕ



Strain $= \varepsilon_i = \phi y_i$

Stress $= \sigma_i = E \phi y$

Force $= F_i = E \phi y_i A_i$

Moment About Centroid = Force · Lever Arm

Moment Component $= E \phi y_i A_i y_i$

Total Moment $= \sum_{i=1}^{n} E \phi y_i^2 \cdot A_i$

take limit as $A_i \to 0$

$$M = \int E\phi \cdot y^2 dA$$

$$= E\phi \underbrace{\int y^2 dA}_{\text{Second Moment of Area} = I}$$

$$M = E \phi I$$

$$\boxed{M = EI\phi}$$